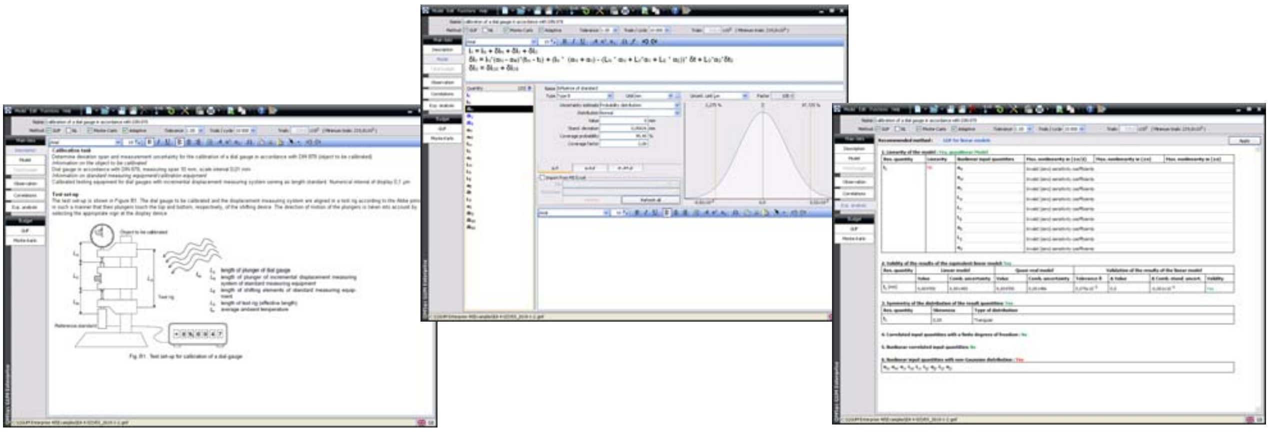
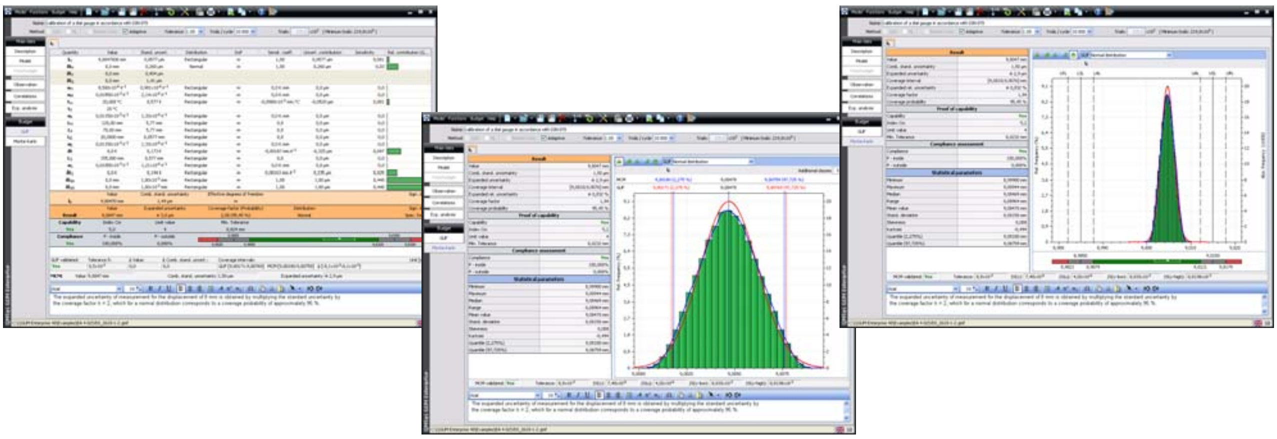


QMSys GUM
Enterprise, Professional, Standard
Software Tools for Measurement Uncertainty Analysis



QMSys Metrology and measurement software

1. Introduction

QMSys GUM Software products are comprehensive tools for analysis of the measurement uncertainty of physical measurements, chemical analyses and calibrations. Whether you are a scientist, metrologist, design engineer, production engineer, test engineer or anyone dealing with measurement accuracy, you need to know only the information that falls within your technical specialty. Our products are the ultimate assistant to your practice, refining it with proven professionalism and reliability. ***QMSys GUM Software*** furnishes the statistical analysis, while you furnish the technical knowledge.

The software complies with numerous international guidelines and standards. Some of the most recognized ones are:

- **ISO/IEC Guide 98-3:2008 (GUM:1995)** Guide to the expression of uncertainty in measurement
- **ISO/IEC Guide 98-3:2008/Suppl. 1:2008** Supplement 1 to the "GUM" - Propagation of distributions using a Monte-Carlo method
- **ANSI/NCSL Z540.2** U.S. Guide to the Expression of Uncertainty in Measurement
- **EA-4/02** Expression of the Uncertainty of Measurement in Calibration
- **DAkkS-DKD-3** Expression of the Uncertainty of Measurement in Calibration
- **UKAS M3003** Expression of Uncertainty and Confidence in Measurement
- **EURACHEM/CITAC Guide CG 4** Quantifying Uncertainty in Analytical Measurement
- **VDA Band 5** Measuring Process Suitability
- **ASME PTC 19.1-2005** Test Uncertainty
- **ISO 14253-1** - Decision rules for proving conformance or non-conformance with specifications
- **ANSI B89.7.3.1** - Guidelines for decision rules: Considering measurement uncertainty in determining conformance to specifications
- **EURACHEM** - Use of uncertainty information in compliance assessment ISO/IEC/EN 17025

The software uses three different methods to calculate the measurement uncertainty:

- **GUF Method for linear models** - this method is applied to linear and quasi-linear models and corresponds to GUM Uncertainty Framework. The software calculates the partial derivatives (the first term of a Taylor series) to determine the sensitivity coefficients of the equivalent linear model and then calculates the combined standard uncertainty in accordance with the Gaussian error propagation law.
- **GUF Method for nonlinear models** - this method is provided for nonlinear models with symmetric distribution of the result quantities. In this method, a series of numerical methods are used – e.g. nonlinear sensitivity analysis, second and third order sensitivity indices, quasi-Monte Carlo with Sobol sequences. The additional influences, such as non-linear relationships, correlations, distribution type or interaction of the input quantities, are also taken into account when calculating the uncertainty contributions. The results obtained with this method coincide with the analytical method remarkably closely.
- **Monte-Carlo Method** - this method is described in the first supplement to GUM and it is the only appropriate method for many calculations of the uncertainty, since the equations of the model are often not linear. In the Monte Carlo technique, a suitable distribution is attributed to each input quantity. From these distributions, a "random value" for each is simulated and a value of the target quantity is calculated from this set of input data. This procedure is repeated many times, so that a set of data are obtained for the result quantity, which represents a random sample from the "potential" values of the result quantity as a function of variations in the input quantities according to their distribution. The mean value and the standard deviation of this random sample are estimates for the value of the result quantity and its standard uncertainty. In order to achieve reliable estimates, a high number of replicates are necessary - usually from $2x10^5$ up to 10^6. The Monte Carlo technique, however, provides far more than an estimate for the result quantity and its standard uncertainty, namely: an estimated distribution of the result quantity and a realistic coverage interval.

Using these methods the ***QMSys GUM Software*** offers plausible and accurate calculation of the measurement uncertainty for virtually all types of measurements:

- linear and nonlinear models
- symmetric and asymmetric distributions of the result quantities
- correlated input variables with arbitrary probability distribution

A special algorithm for generation of correlated values for specified probability distributions is developed and implemented in the program in order to ensure the accuracy and validity of the results even in cases, that are not described in GUM and GUM Supplement 1:

- Correlated non-normally (non Gaussian) distributed input quantities
- Correlated input quantities with finite degrees of freedom
- Nonlinear models with more than two correlated input quantities with arbitrary distribution.

The program supports the systematic procedure in building an uncertainty analysis, as requested in the corresponding standards and guides. This process consists of the following basic steps:

- Creation of mathematical model, which describes the relationship between the quantities in the respective measurement
- Analysis of the required information as the standard measurement uncertainty or the distribution of values of input quantities
- Entry of the observations
- Determination of the correlation coefficient between the input quantities
- Analysis of the model and selecting the appropriate method for calculating the measurement uncertainty
- Calculation of the measurement uncertainty and preparation of the measurement uncertainty budget
- Validation of the results – estimate, combined uncertainty and coverage interval (expanded uncertainty)

The computation examples in the documents **GUM, GUM Supplement 1, EA-4/02, DAkkS-DKD-3** and **EURACHEM/CITAC Guide CG 4** are added to the software package as example models that can be analyzed with the program.

The result of the evaluation is a clearly structured measurement uncertainty budget in a table form. This table holds all used quantities with their quantity names and values, the associated standard uncertainty and effective degrees of freedom, the sensitivity coefficient automatically derived from the model equation and the contribution to the standard uncertainty of the result of the measurement. Finally, the complete result of the examination is presented as a value with associated expanded uncertainty and automatically or manually selected coverage factor.

The Monte Carlo method displays a histogram, statistical parameters of the estimated distribution of the result quantities and validation of the results. For result quantities with asymmetric distribution, the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty and asymmetric coverage factor.

The summary budget offers the following additional analysis:

- Correlation analysis of the result quantities
- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range.

The result of the uncertainty analysis together with all input data can be printed with the help of configurable templates as a report. All input texts are part of the printout and are used for documentation purposes.

Each analysis can be completely saved in a file with a selectable name. In this way, the examination is available at any time for a later review or editing. Each saved analysis can be used as a starting point for new uncertainty analyses using the same model, but with new and changed data.

Validation of the QMSys GUM Software

QMSys GUM Software is a standard application that offers the possibility for the user to freely enter or modify the model equation. With this feature, the application can be used to evaluate almost any measurement process. Therefore, a general validation by ***Qualisyst Ltd.*** for all possible purposes is not possible. The correct calculation can be verified with the help of the examples from the official regulations and guidelines (**GUM, Supplement 1 to the GUM, EA-4/02, DAkkS-DKD-3 and EURACHEM/CITAC Guide CG 4**), which are part of the installation. These documents are available for free downloading. The results of the validation of the ***GUM QMSys software*** are shown in the tables in Appendix A.

2. Editions of the QMSys GUM Software

- **GUM Enterprise** provides the highest-precision analysis of the measurement uncertainty for all types of measurements
- **GUM Professional** offers accurate analysis of measurement uncertainty for linear and nonlinear models with symmetric or asymmetric distribution of the output quantities
- **GUM Standard** offers calculation of measurement uncertainty for linear and nonlinear models with symmetric distribution of the output quantities

The functionality of the different versions is shown in the following tables:

Key Features

Function	GUM Enterprise	GUM Professional	GUM Standard
Modelling the measurement	Yes		
Several output quantities	Yes		
Expert analysis	Yes		
GUM Method for linear models (GUF)	Yes		
GUM Method for nonlinear models (GUF-NL)	Yes		
Monte-Carlo Method	Yes		No
Calculation of nonlinear uncertainty contributions	Yes		
Suitable for linear and quasi-linear models	Yes		
Suitable for non-linear models	Yes, symmetric and asymmetric distribution of the output quantities		Yes, symmetric output quantities
Proof of capability and compliance assessment	Yes, several capability indexes and decision rules		
Uncertainty budget	Yes		
Extended analysis of several output quantities	Yes		No
Reports and export	Yes		
Server - version	Yes, concurrent user licenses, unlimited number of all users		
Portable version on a USB memory stick	Yes		

Modelling of the measurement process

Function	GUM Enterprise	GUM Professional	GUM Standard
Free definable model equation	Yes		
Number of the input and output quantities	unlimited		
Measurement units catalogue	Yes		
Correlation matrix of the input variables	Yes		
Validation of the correlation matrix	Yes		
Optimizing of the correlation matrix	Yes		No
Type A Input Quantities	**Yes**		
Method of Observation	• Direct - individual values or group values • Indirect - free definable measurement cycles		
Number of observations	unlimited		
Data import via clipboard	Yes		
Import from Microsoft Excel - files	Yes		No
Determining of the standard uncertainty	• Experimental or a pooled estimate • Normal or t-distribution • Standard uncertainty or standard deviation • Bayesian standard uncertainty with t-distribution		
Correlation analysis of the observations	Yes		
Statistical analysis, histogram	Yes		
Type B Input Quantities	**Yes**		
Estimate of the uncertainty	• Expanded uncertainty with normal or t-distribution • Standard uncertainty with normal or t-distribution • Error limits with rectangular distribution • Probability distribution		
Probability distributions	• Normal distribution • t-distribution • Triangle distribution • U-shaped distribution • Exponential distribution • Cosine distribution	• Log-normal distribution • Rectangular distribution • Trapezoidal distribution • Curvilinear trapezoidal distribution • Square distribution • Half-Cosine distribution	
Entering the distribution parameters	• Value and standard uncertainty • Value and half-width of the distribution area • Lower and upper limits		
Relative uncertainty error	Entered in %, calculating the degrees of freedom acc. to GUM, G.3		
Import from Microsoft Excel - files	Yes		No

Expert Analysis

Function	GUM Enterprise	GUM Professional	GUM Standard
Linearity test	Yes, calculated in six points for each input variable		
Validation of the results	Yes, value and combined standard uncertainty		
Analysis of the probability distribution of the output quantities	Yes, symmetry and type of the probability distribution		
Checking for correlated input quantities with a finite degree of freedom	Yes		
Checking for non-linear correlated input quantities	Yes		
Checking for non-linear non-normally distributed input quantities	Yes		

Methods for calculation of the measurement uncertainty

Function	GUM Enterprise	GUM Professional	GUM Standard
GUM Method for linear models (GUF)	**Yes**		
Calculation of sensitivity coefficients	Yes		
Calculation of the effective degrees of freedom	Yes		
Calculation of the expansion factor for any coverage probability	• Normal distribution • Rectangular distribution • Trapezoidal distribution • t-distribution • Triangle distribution • Other symmetric distributions		
Expanded uncertainty	Yes		
Validation of the GUF Method	Yes		Yes, in the expert analysis window
GUM Method for nonlinear models (GUF-NL)	**Yes**		
Non-linear sensitivity analysis	Yes		
Sensitivity indices of higher order	Yes, up to 3rd order	Yes, 2nd order	
Calculation for correlated input variables	Yes, all distribution types		
Calculation of the effective degrees of freedom	Yes, even for correlated input variables		
Calculation of the expansion factor for any coverage probability	• Normal distribution • Rectangular distribution • Trapezoidal distribution • t-distribution • Triangle distribution • Other symmetric distributions		
Expanded uncertainty, coverage interval	Yes		
Validation of the GUF-NL Method	Yes		No
Monte-Carlo Method	**Yes**		**No**
Number of simulations	10^4 to 10^8	10^4 to 10^6	-
Random number generators*	CMWC4096, Mersenne Twister, enh. Wichmann/Hill, ...	enh. Wichmann/Hill	-
Adaptive Monte-Carlo Procedure	Yes	No	-
Calculation for correlated input variables	Yes, all distribution types		-
Validation of the Monte-Carlo method	Yes	No	-
Recognition of the distribution of the resulting values	Yes	No	-
Expanded uncertainty, coverage interval	Yes, even for asymmetric distributions		-

* Period: CMWC4096 by Dr. Marsaglia - 6,58*10^{39460} ; Mersenne Twister - 4,32*10^{6001} ; verb. Wichmann/Hill - 2,63*10^{36}

Evaluation of the results

Function	GUM Enterprise	GUM Professional	GUM Standard
Uncertainty budget	Yes		
Sensitivity, relative contribution and Pareto diagram	Yes		
Statistical analysis of the resulting quantities	Yes		No
Statistical analysis of the input quantities	Yes	No	Nein
Proof of capability	Yes, several capability indexes		
Compliance assessment	Yes, multiple decision rules, graphic of the conformity zones		
Documentation fields	Yes		
Inclusion of images and graphics	Yes		
Extended Analysis of Several Output Quantities	**Yes**		**No**
Summary table of the resulting quantities	Yes		-
Regression analysis - calculation of the equation of measurement uncertainty for a particular range	Yes		-
Correlation analysis of the resulting quantities	Yes		-
Print and Export	**Yes**		
Custom templates	Yes		
Export to clipboard	Yes		
Export to Microsoft Excel	Yes		No

3. Advantages of the QMSys GUM Software

The GUM QMSys software is the only specialized software for uncertainty analysis, which offers the following functionalities:

- Plausible and accurate calculation of measurement uncertainty for practically all types of measurements:
  - linear and nonlinear models
  - symmetric and asymmetric distributions of the result quantities
  - correlated input quantities with non-normal probability distribution
- Consistency of the results with the reference samples in GUM, Supplement 1 to the GUM, EA-4/02, DAkkS-DKD-3, EURACHEM/CITAC Guide CG 4 and other standards and guidelines
- Three different methods to analyse the measurement uncertainty:
  - GUF method for linear and quasi-linear models
  - GUF method for nonlinear models with symmetric distribution of the result quantities
  - Monte-Carlo method for practically all models
- The largest number of probability distributions for input variables
- User-definable measurement cycles for input quantities of type A
- Multiple ways to enter parameters of the probability distribution for input quantities of type B:
  - Value and standard deviation or standard uncertainty
  - Value and half-width of the limits of the distribution interval
  - Limits of the distribution interval
- Definition of the relative uncertainty of the estimate of the measurement uncertainty for input quantities of type B and calculating the degrees of freedom
- Any coverage probability of the result quantities, not just 95%
- Expert analysis of the model that examines the conditions for the application of each method and determines the appropriate methods for the calculation of measurement uncertainty
- Calculation of the uncertainty contributions with the GEF-NL method for nonlinear models for:
  - non-linear relationships between input quantities and result quantities
  - simultaneous interaction of multiple input quantities
- The best random number generators for carrying out the Monte-Carlo simulations - CMWC4096 (Complementary-Multiply-With-Carry) by Dr. Marsaglia and Matsumoto's Mersenne Twister
- Monte-Carlo simulations also for models with correlations between non-normally distributed input quantities
- Automatic fitting of the probability distribution of the result quantities and calculation of the distribution parameters from the Monte-Carlo simulations
- Automatic calculation of the coverage factor of the result quantities for the selected probability distribution and the specified coverage probability
- Validation of the results of the Monte-Carlo method
- Validation of the results of the GUF method with the help of the Monte-Carlo method
- Calculating the expanded uncertainty for asymmetric distribution of the result quantities
- Proof of capability and compliance assessment with several decision rules acc. to ISO 14253-1, ANSI B89.7.3.1
- Regression analysis and calculation of the equation of the expanded uncertainty for a particular range
- Complete information on the sources of uncertainty and a better understanding of the measurement process
- Approaches for optimization of the measurement procedures and minimizing the uncertainty of new and existing measurement processes
- The user can create new or adapt existing templates for the uncertainty reports by using a text editor like Microsoft Word®
- 30-day trial period for all editions of the ***QMSys GUM software***.

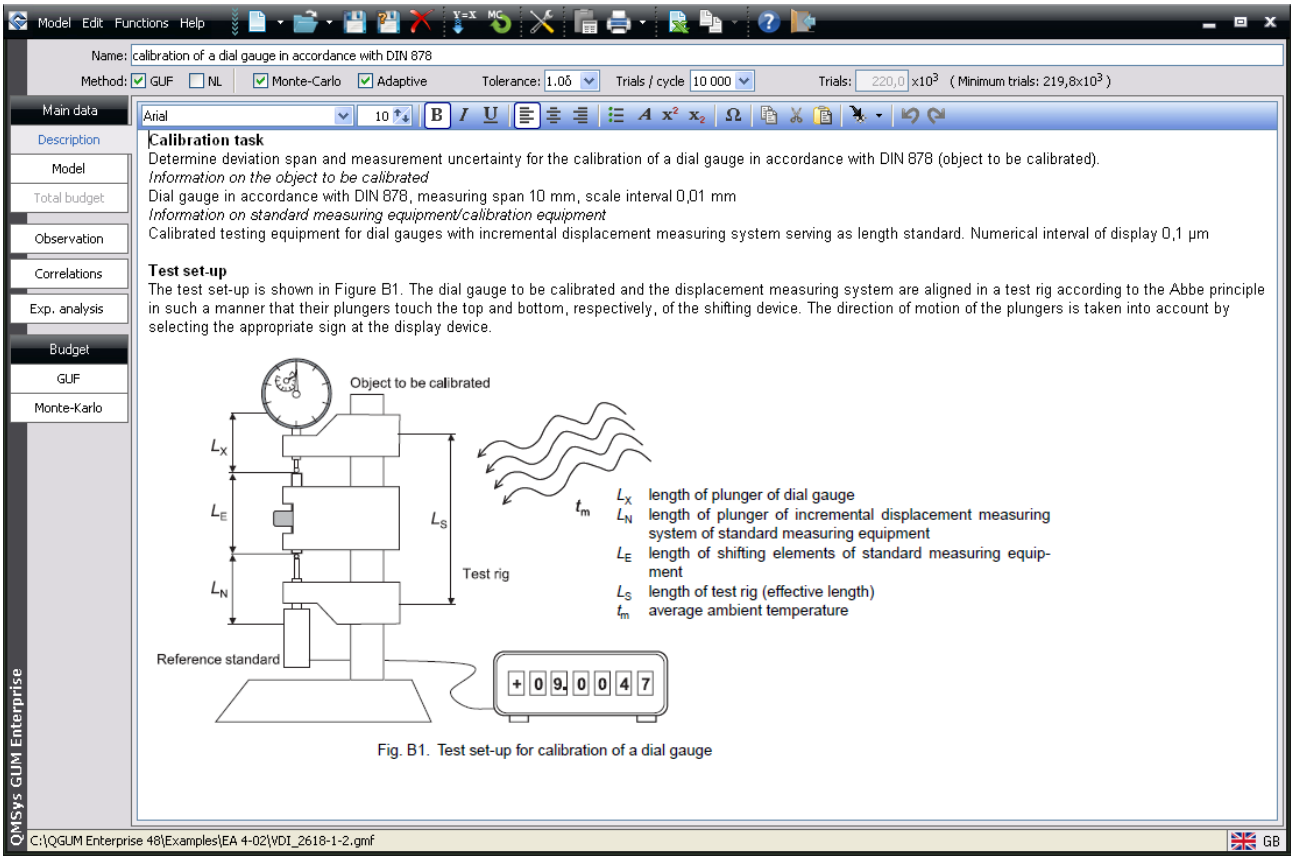
4. Software Description

The graphical user interface of the *QMSys GUM software* is based on several views, which are further structured by dialog pages. The title of the project and the settings for the methods for calculating the measurement uncertainty are entered in the upper range of the program window.

The following data are positioned on separate views:

- ***Main data*** - the registers *Description*, *Model* and *Total budget* are available. By selecting the different registers, the corresponding data can be viewed or edited.
- ***Observation*** – this view processes the values of type A quantities.
- ***Correlations*** – known correlations between the input quantities are entered in a matrix of correlation coefficients.
- ***Expert analysis*** - the software performs an advanced analysis of the model and determines the appropriate methods for the following calculation of the uncertainties.
- ***Budget*** – this view presents the results of the analysis.

The program menu and the toolbar provide specific functions for the selected view.



4.1. Modelling the measurement process

4.1.1. Register Description

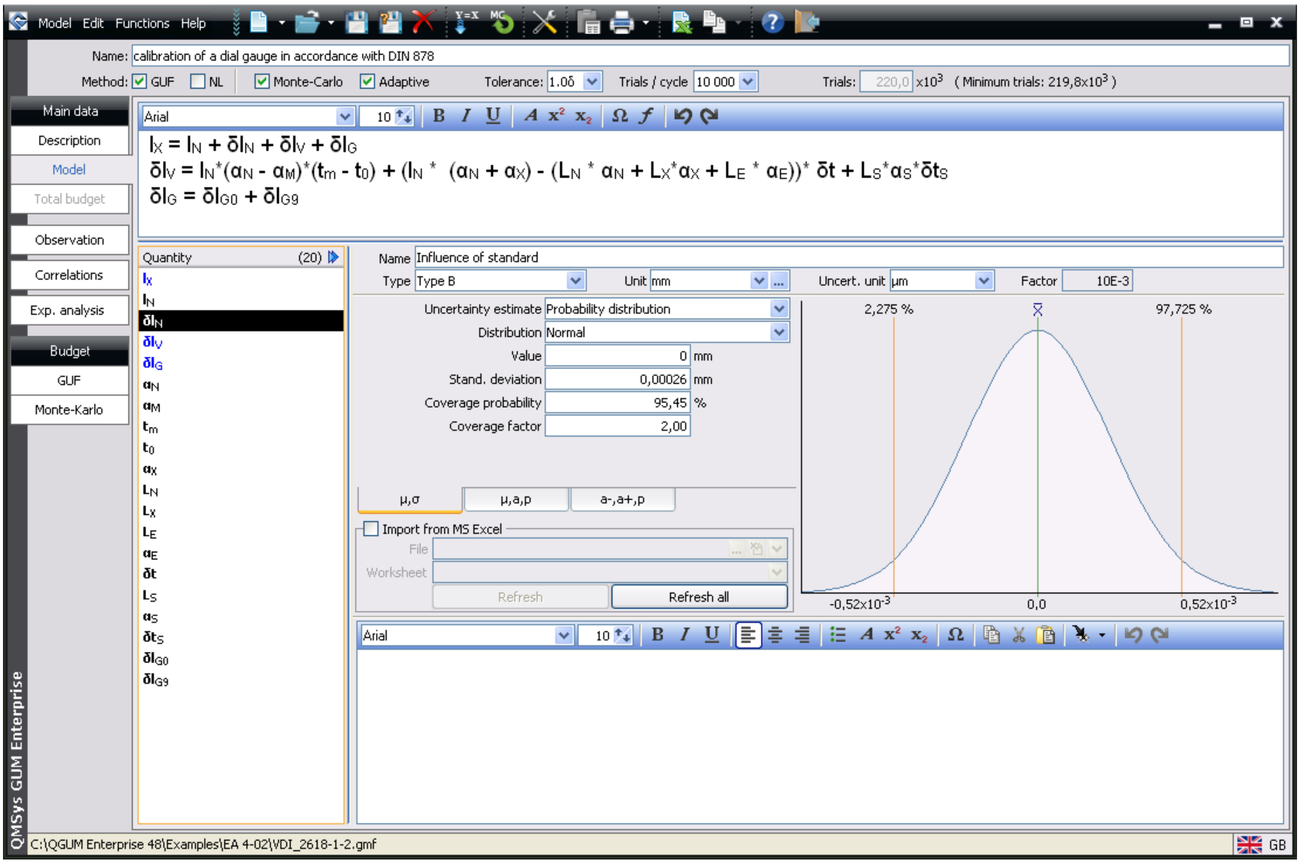
In the ***Description*** register page, a general description of the measurement task can be entered. This data are used for informational purposes and is part of the printout.

Images can be paste from the clipboard or with the *Insert object* button in the toolbar. Other objects (files) can also be imported and saved in the file of the uncertainty analysis. Double click on the inserted object will start the appropriate program to view or edit the object.

4.1.2. Register Model

The dialog page ***Model*** in the view ***Main data*** holds the mathematical model of the uncertainty analysis and the specification data of all quantities.

In the upper field on this page, the user enters the equations of the mathematical model. The model equations are the starting point for all subsequent calculations by the software. It is always possible to insert new quantities into the equation, and to rename or to delete existing quantities. Additional functions are available in a toolbar above the equation field.

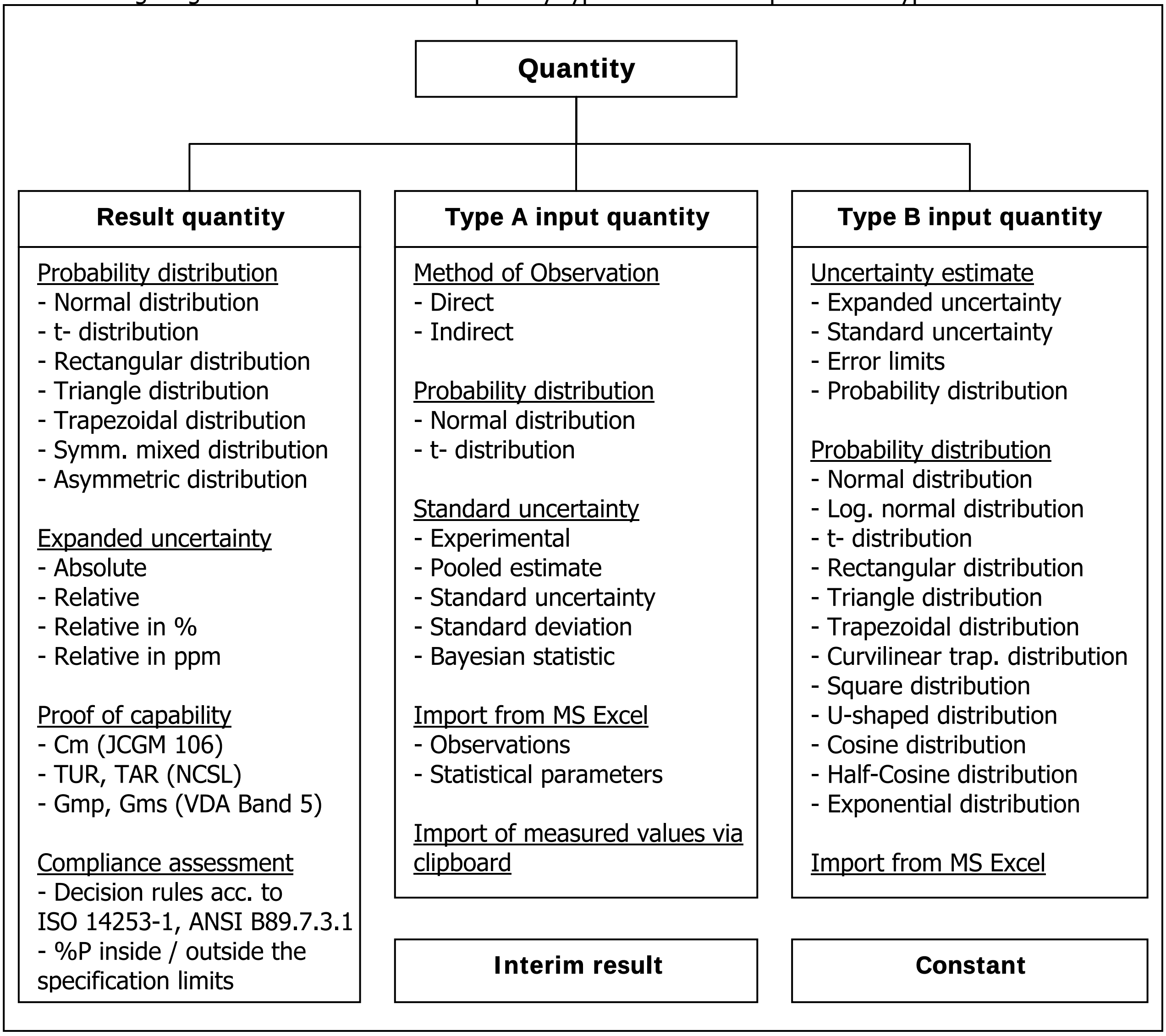


When a model equation is more complex and contains a large number of input quantities, it is advisable to split it into smaller parts by introducing interim results, and thereby to make it easier to understand.

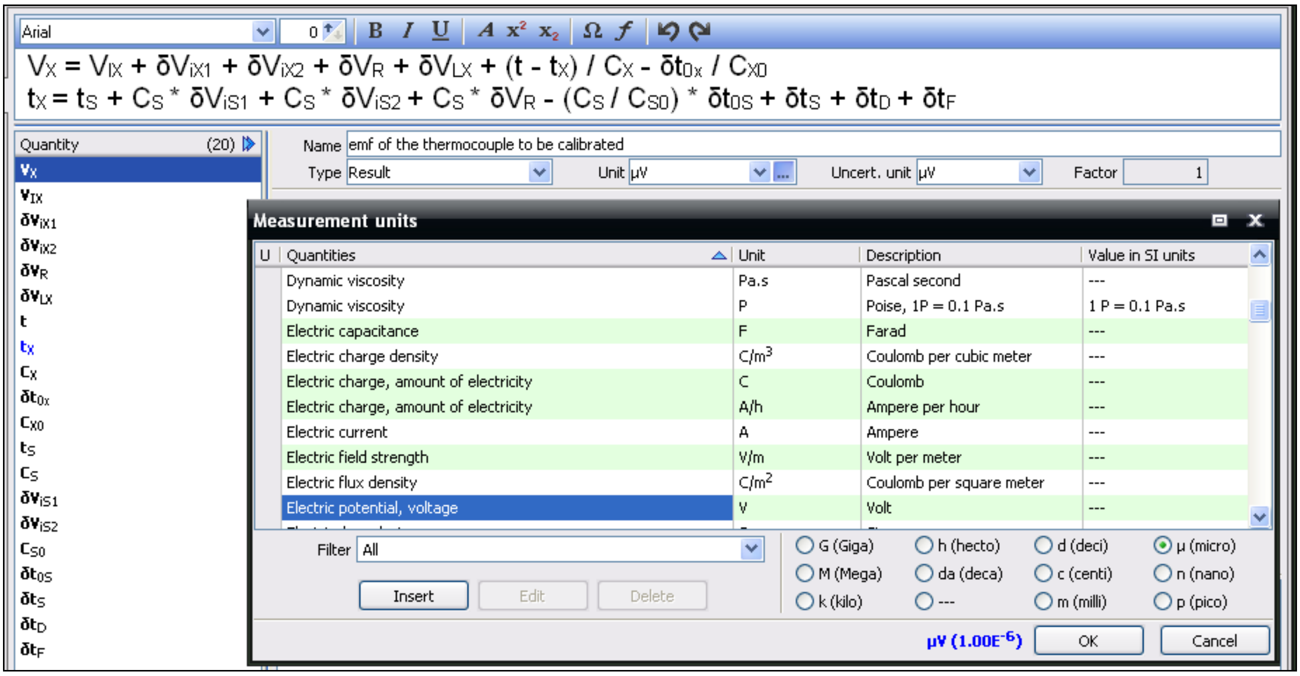
After a new entry or a change in the mathematical model, the syntax of the equations is checked and the list of quantities is rebuilt or updated. The current data of the selected quantity are displayed to the right and may be edited. The following table provides an overview of the different quantity types.

Type	Description	Comment
Result	Measurand	This type identifies the output quantity and is set automatically by the program.
Interim result	Internal measurand	The program sets this type automatically. Switching to the type "Result" is possible.
Type A	Repeatedly observed quantity	The value and the standard uncertainty of the quantity are evaluated by using statistical analysis of measurement series. Optionally an estimate of the standard uncertainty can be specified.
Type B	Not-repeatedly observed quantity	The value and the standard uncertainty of the quantity are evaluated using means other than statistical analysis of measurement series. For these quantities, the appropriate distribution is selected and parameterized.
Constant	Mathematical constant	For mathematical constants without uncertainty, only the value can be entered.

The following diagram shows the different quantity types with their respective subtypes.



A basic unit of the quantity value and additional unit for the measurement uncertainty can be assigned to every quantity in the model. The program provides an adequate database with SI units and some other commonly used units outside the SI. New custom measurement dimensions and units can be added to the database.

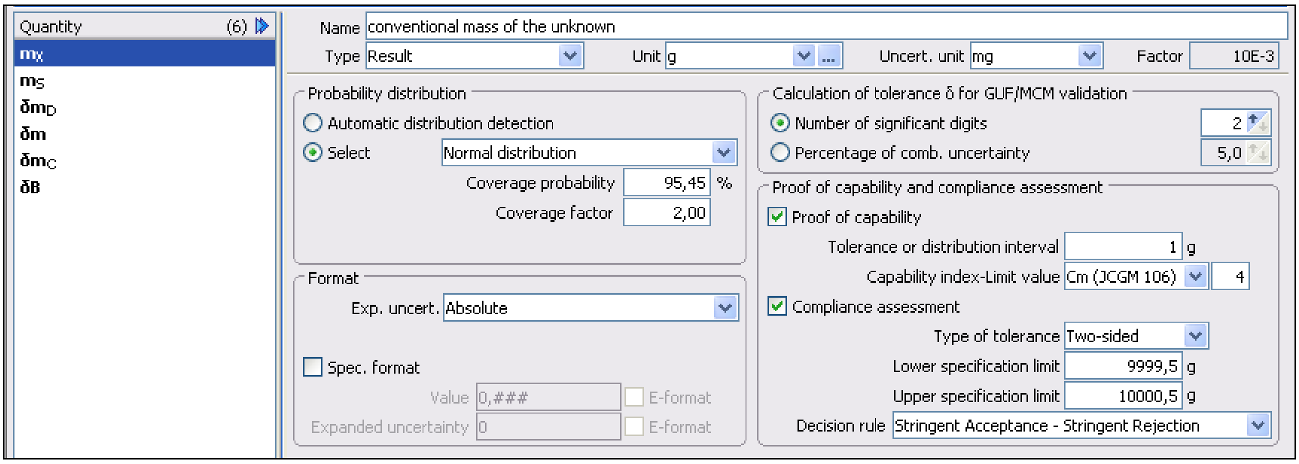


4.2. Proof of Capability and Compliance Assessment

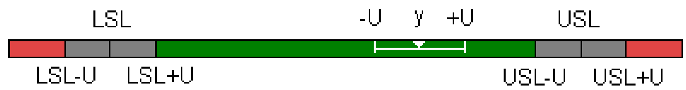
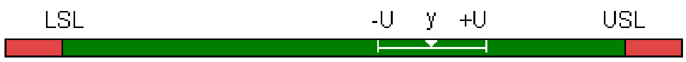
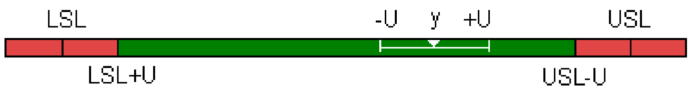
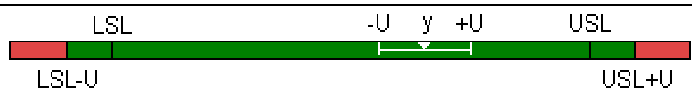
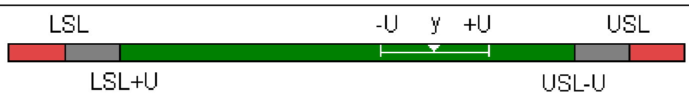
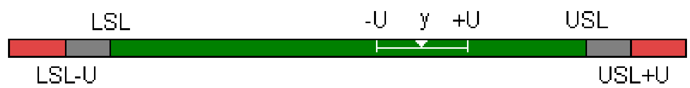
The basic approach is to set the uncertainty in relation to the tested tolerance and to use this relation as an evaluation criterion. The program offers several capability indexes for evaluating the capability of measuring systems and measurement processes:

Reference	Capability Index
JCGM 106	Cm (Measurement capability index)
NCSL Glossary of Metrology related Terms	TUR (Test Uncertainty Ratio)
	TAR (Test Accuracy Ratio)
VDA Volume 5	Gmp (Measurement process capability)
	Gms (Measuring system capability)

In order to classify measuring systems and processes, the software calculates the minimum tolerance, at which the measuring system or the measurement process is currently still capable.



The software offers several decision rules for compliance assessment of the measurement results in accordance with the standards ISO 14253-1, ANSI B89.7.3.1 and other guides:

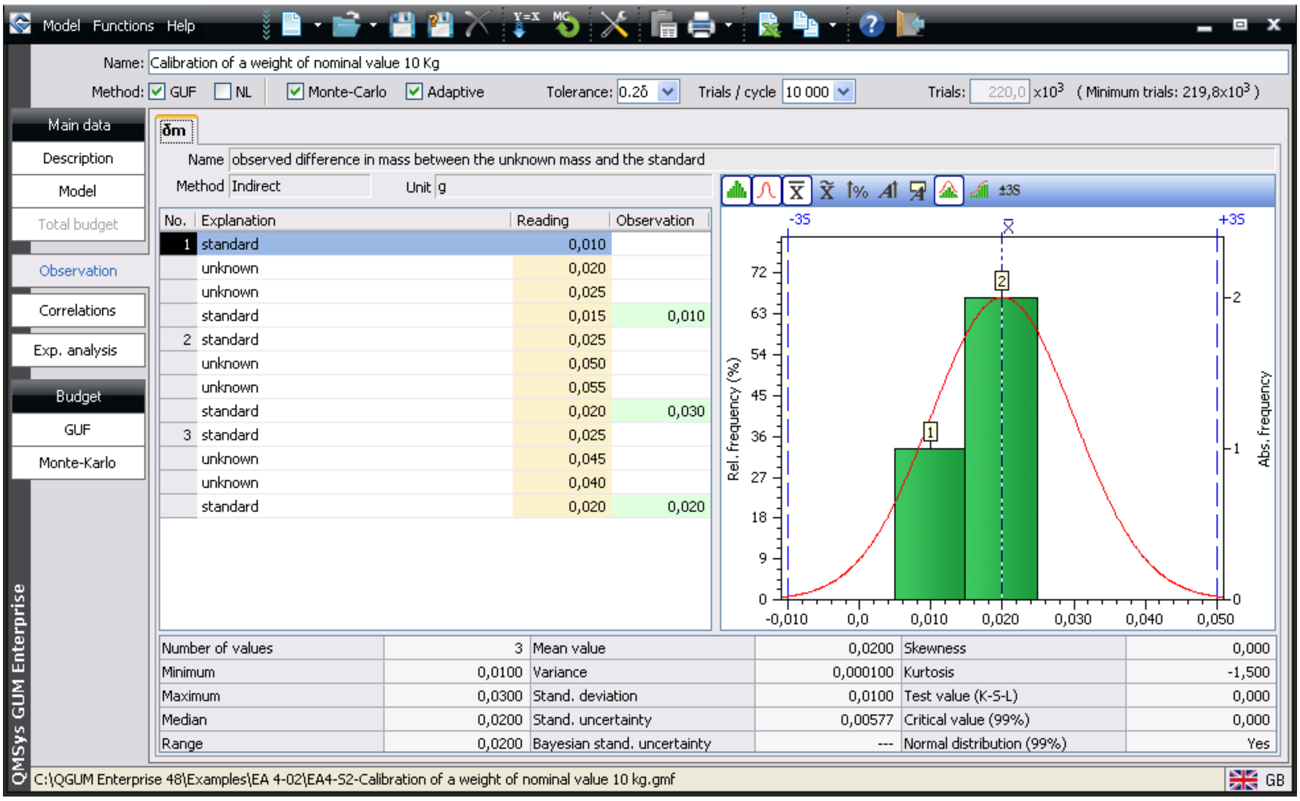
Decision Rule	Acceptance, Rejection and Uncertainty Zones *
Stringent Acceptance - Stringent Rejection (Standard rule acc. to ISO 14253-1, ANSI B89.7.3.1, VDA Volume 5)	
Simple Acceptance - Simple Rejection (Measurement uncertainty is not taken into account when determining the acceptance and rejection zones)	
Stringent Acceptance - Relaxed Rejection (higher cost of accepting a nonconforming product and lower product costs)	
Relaxed Acceptance - Stringent Rejection (higher product costs and lower cost of accepting a nonconforming product)	
Stringent Acceptance - Simple Rejection (medium cost of accepting a nonconforming product and lower product costs)	
Simple Acceptance - Stringent Rejection (medium product costs and lower cost of accepting a nonconforming product)	

*LSL – lower specification limit, USL – upper specification limit, y – result, U – expanded MU.

For a better estimate of the conformity, the software calculates the actual probability that the result is inside or outside the specification limits.

4.3. Entering and Import of Observation Data

The view ***Observation*** processes the values of repeatedly observed quantities. The data is typed into a table, the structure of which depends on the method of observation. All readings and observations must be given in the same unit, as the one entered for the quantity value. If the model uses several type A quantities, the observed quantity can be selected at the upper border of the window.

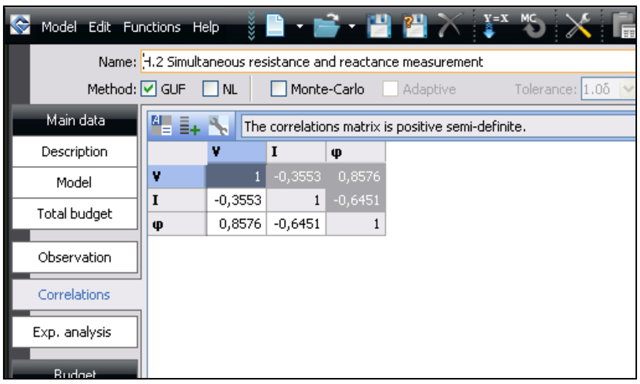


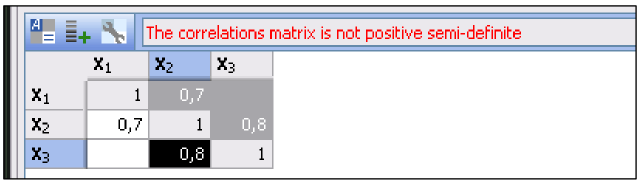
The data for an observed quantity of type A can be imported from the clipboard. The software editions ***GUM Enterprise*** and ***GUM Professional*** offers also import of measured values from a Microsoft Excel® file. The data are read in, checked and inserted in the observation table.

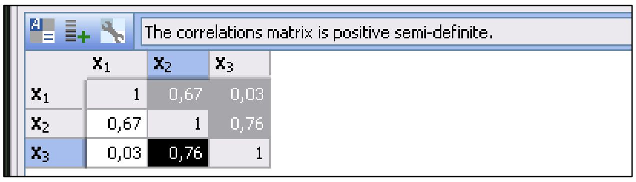
When valid data for all observations (or readings) have been entered, statistical information including mean value, standard deviation, standard uncertainty and Histogram of the data will be displayed.

4.4. Analysis and Entering of Correlations between the Input Quantities

In the ***Correlations*** page, known correlations between the input quantities are entered in a matrix of correlation coefficients. The input quantities of type A can be analysed for possible correlation between measurands. Further input quantities can be inserted in the correlation matrix.







The software checks automatically if the correlation matrix is positive semi-definite. All Eigen values of the positive semi-definite correlation matrix are non-negative (>= 0). If the correlation matrix is not positive semi-definite, it is recommended to perform an optimization with the build in ***Optimization*** function.

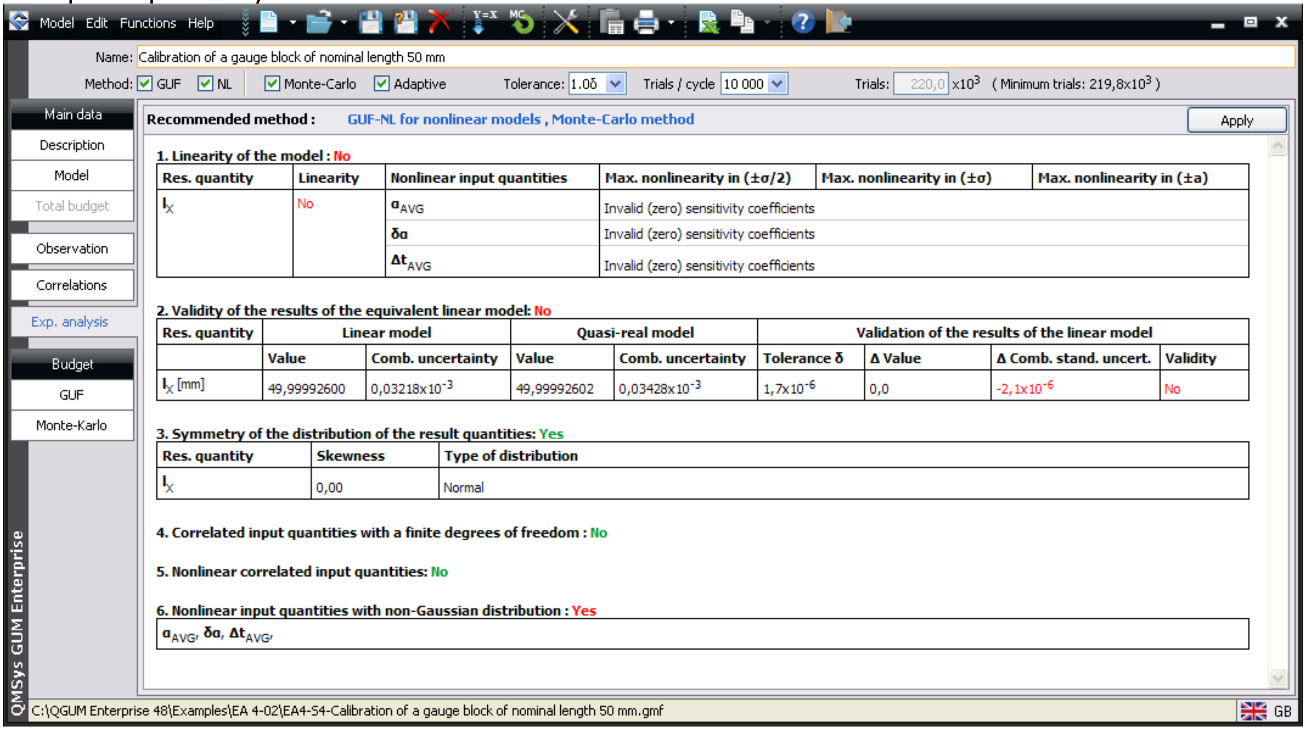
4.5. Expert Analysis of the Model

The view ***Expert analysis*** presents the results of the expanded analysis of the model. The software checks the conditions for the application of the different methods and determines the appropriate methods for the following calculation of the measurement uncertainty.

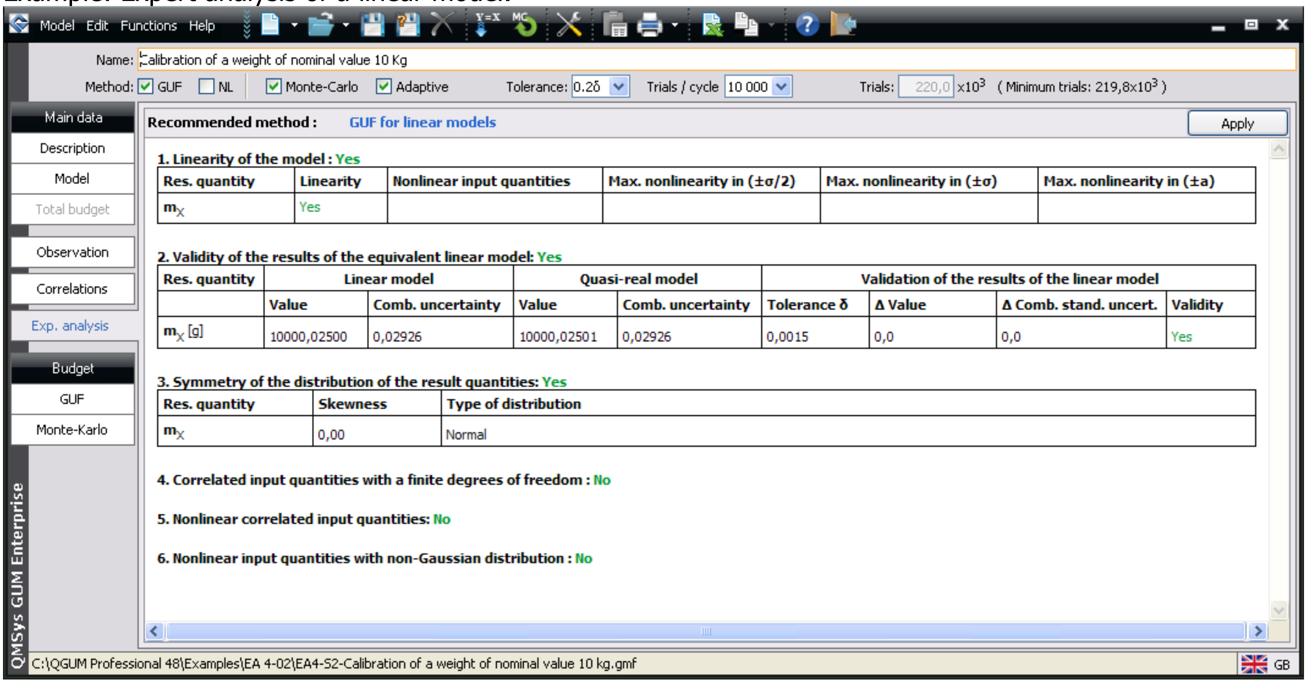
The following tests and calculations are performed:

- Linearity test for each input quantity in sixth areas of the distribution interval
- Calculation of the results of the equivalent linear model and the quasi-real model
- Validating the results of the equivalent linear model (value and combined standard uncertainty)
- Analysis of the distribution of the result quantities, determination of the symmetry and the distribution type
- Check for correlated input quantities with a finite degree of freedom
- Check for non-linear correlated input quantities
- Check for non-linear non-normally distributed input quantities.

Example: Expert analysis of a nonlinear model.



Example: Expert analysis of a linear model.

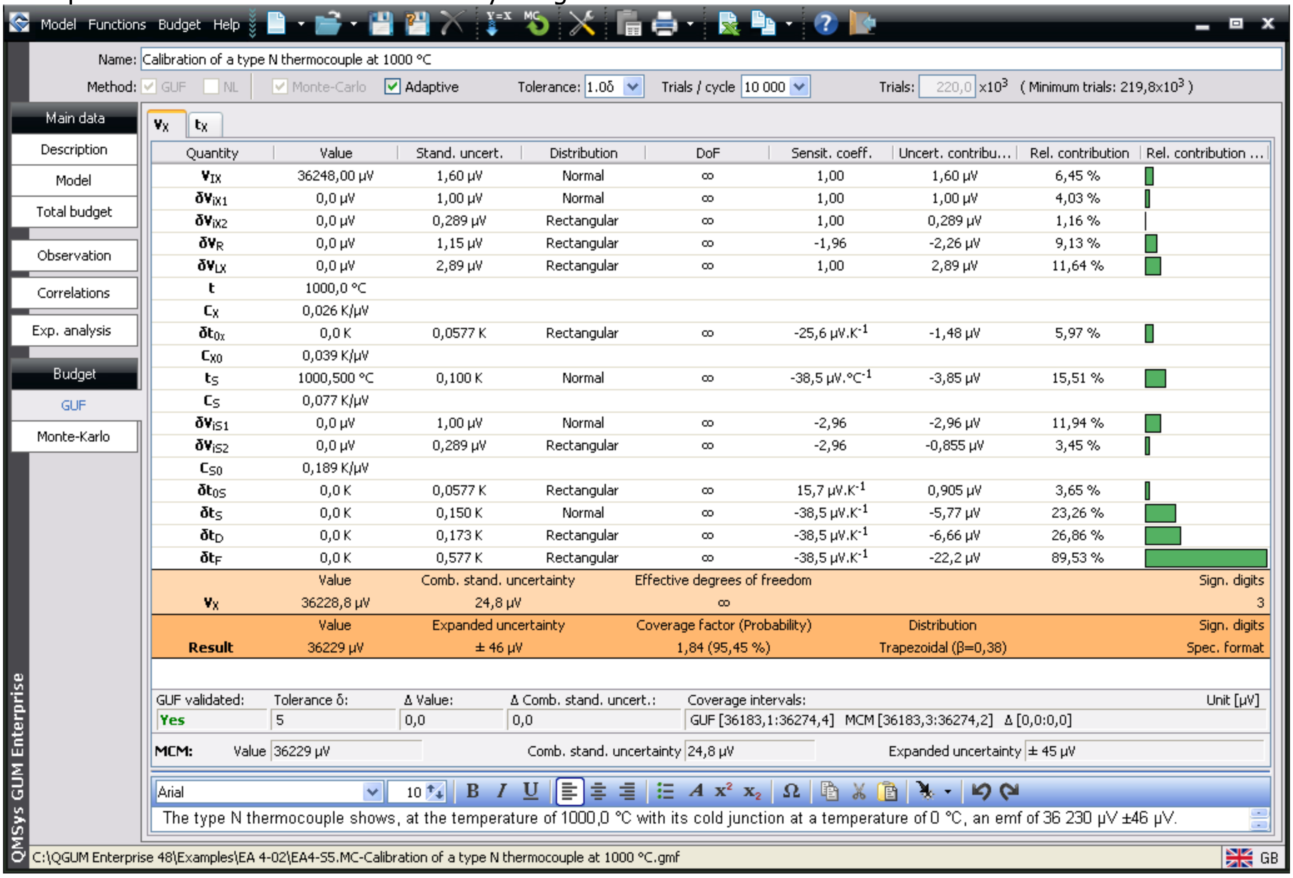


4.4. Measurement Uncertainty Budgets

The result of the analysis is presented in pages *GUF* and *Monte-Carlo* of the *Budget* view.

The page *GUF* shows a clearly structured measurement uncertainty budget in a table form. This table holds all used quantities with their quantity names and values, the associated standard uncertainty and effective degrees of freedom, the sensitivity coefficient automatically derived from the model equation and the contribution to the standard uncertainty of the result of the measurement. The *Interim results* are only shown with the value and the standard uncertainty. Additional columns can be activated in the *Budget* menu.

Example: GUF – Measurement uncertainty budget



Quantity	Value	Stand. uncert.	Distribution	DoF	Sensit. coeff.	Uncert. contribu...	Rel. contribution
V_{IX}	36248,00 µV	1,60 µV	Normal	∞	1,00	1,60 µV	6,45 %
δV_{iX1}	0,0 µV	1,00 µV	Normal	∞	1,00	1,00 µV	4,03 %
δV_{iX2}	0,0 µV	0,289 µV	Rectangular	∞	1,00	0,289 µV	1,16 %
δV_R	0,0 µV	1,15 µV	Rectangular	∞	-1,96	-2,26 µV	9,13 %
δV_{LX}	0,0 µV	2,89 µV	Rectangular	∞	1,00	2,89 µV	11,64 %
t	1000,0 °C						
C_X	0,026 K/µV						
δt_{0X}	0,0 K	0,0577 K	Rectangular	∞	-25,6 µV.K⁻¹	-1,48 µV	5,97 %
C_{X0}	0,039 K/µV						
t_S	1000,500 °C	0,100 K	Normal	∞	-38,5 µV.°C⁻¹	-3,85 µV	15,51 %
C_S	0,077 K/µV						
δV_{iS1}	0,0 µV	1,00 µV	Normal	∞	-2,96	-2,96 µV	11,94 %
δV_{iS2}	0,0 µV	0,289 µV	Rectangular	∞	-2,96	-0,855 µV	3,45 %
C_{S0}	0,189 K/µV						
δt_{0S}	0,0 K	0,0577 K	Rectangular	∞	15,7 µV.K⁻¹	0,905 µV	3,65 %
δt_S	0,0 K	0,150 K	Normal	∞	-38,5 µV.K⁻¹	-5,77 µV	23,26 %
δt_D	0,0 K	0,173 K	Rectangular	∞	-38,5 µV.K⁻¹	-6,66 µV	26,86 %
δt_F	0,0 K	0,577 K	Rectangular	∞	-38,5 µV.K⁻¹	-22,2 µV	89,53 %

	Value	Comb. stand. uncertainty	Effective degrees of freedom		Sign. digits
V_X	36228,8 µV	24,8 µV	∞		3
	Value	Expanded uncertainty	Coverage factor (Probability)	Distribution	Sign. digits
Result	36229 µV	± 46 µV	1,84 (95,45 %)	Trapezoidal (β=0,38)	Spec. format

GUF validated:	Tolerance δ:	Δ Value:	Δ Comb. stand. uncert.:	Coverage intervals:	Unit [µV]
Yes	5	0,0	0,0	GUF [36183,1:36274,4] MCM [36183,3:36274,2] Δ [0,0:0,0]	

MCM: Value 36229 µV; Comb. stand. uncertainty 24,8 µV; Expanded uncertainty ± 45 µV

The type N thermocouple shows, at the temperature of 1000,0 °C with its cold junction at a temperature of 0 °C, an emf of 36 230 µV ±46 µV.

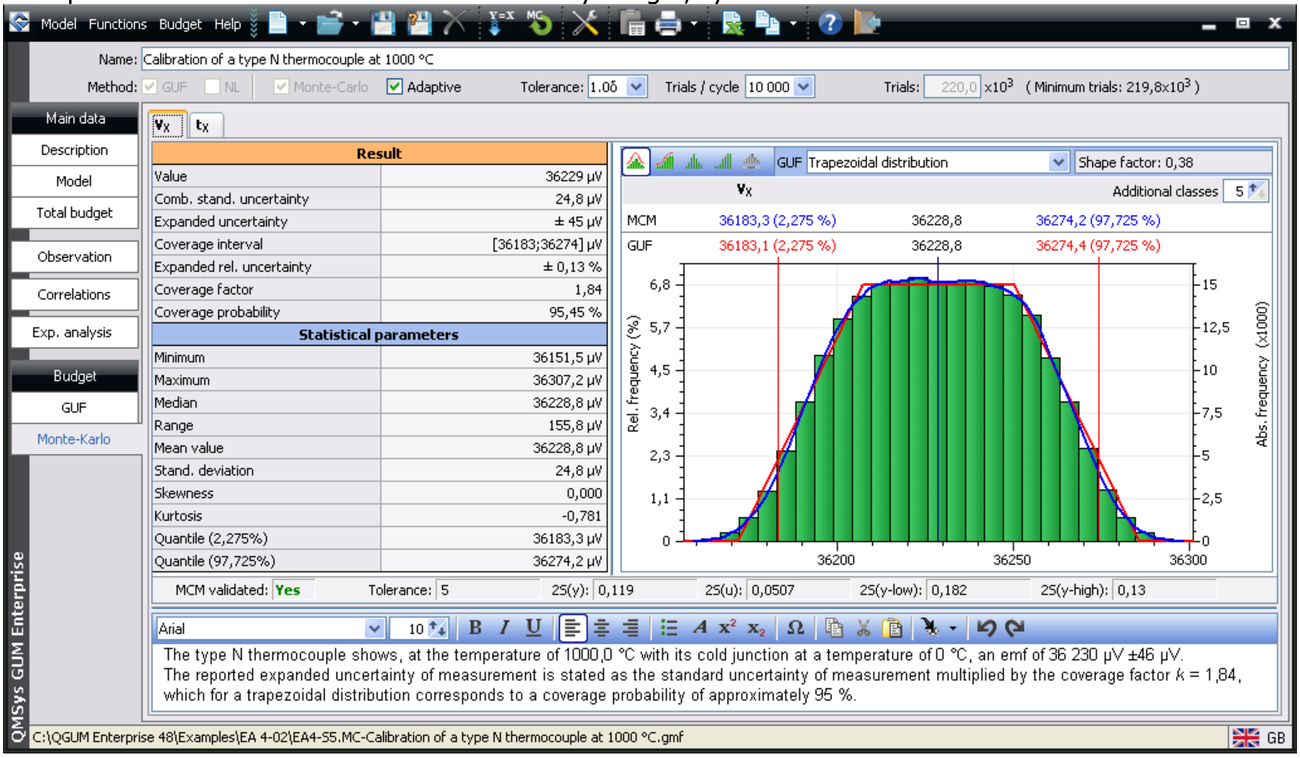
The result quantity is displayed in the bottom line with its value, the corresponding combined standard Uncertainty and the degrees of freedom. Finally, the complete result of the examination is presented as a value with associated expanded uncertainty and automatically or manually selected coverage factor. The results are automatically rounded and displayed in E-Format if necessary.

The *Monte Carlo method* displays a histogram, statistical parameters of the estimated distribution of the result quantities and validation of the results. For result quantities with asymmetric distribution the program estimates the shortest coverage interval, the asymmetric expanded measurement uncertainty and the asymmetric coverage factor.

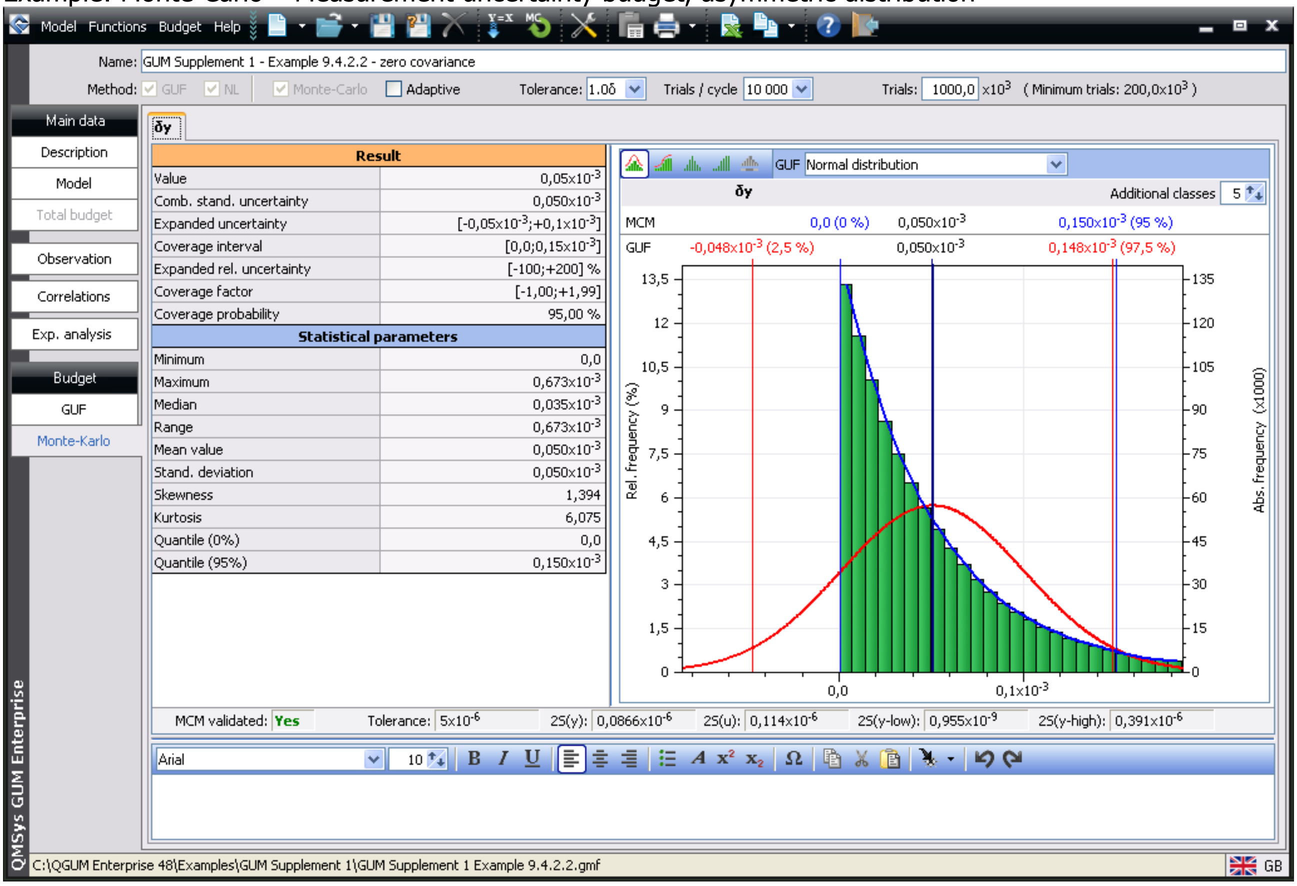
The software automatically validates the results of the GUF Method by comparing the values, the combined standard uncertainties and the limits of the coverage intervals. The numerical tolerance δ in this comparison is calculated based on the combined standard uncertainty and the number of significant digits (2 or 3). The software offers an alternative calculation of the tolerance δ as a percentage of the combined standard uncertainty. Should the comparison be positive, then the GUM uncertainty framework can be used on this occasion and for sufficiently similar models in the future. Otherwise, consideration should be given to using MCM or another appropriate method instead.

The result of the uncertainty analysis together with all input data can be printed with the help of configurable templates as a report. All input texts are part of the printout and are used for documentation purposes. The software editions ***GUM Enterprise*** and ***GUM Professional*** offer a useful feature to export data from an uncertainty analysis over the OLE interface to Microsoft Excel®.

Example: Monte-Carlo – Measurement uncertainty budget, symmetric distribution



Example: Monte-Carlo – Measurement uncertainty budget, asymmetric distribution



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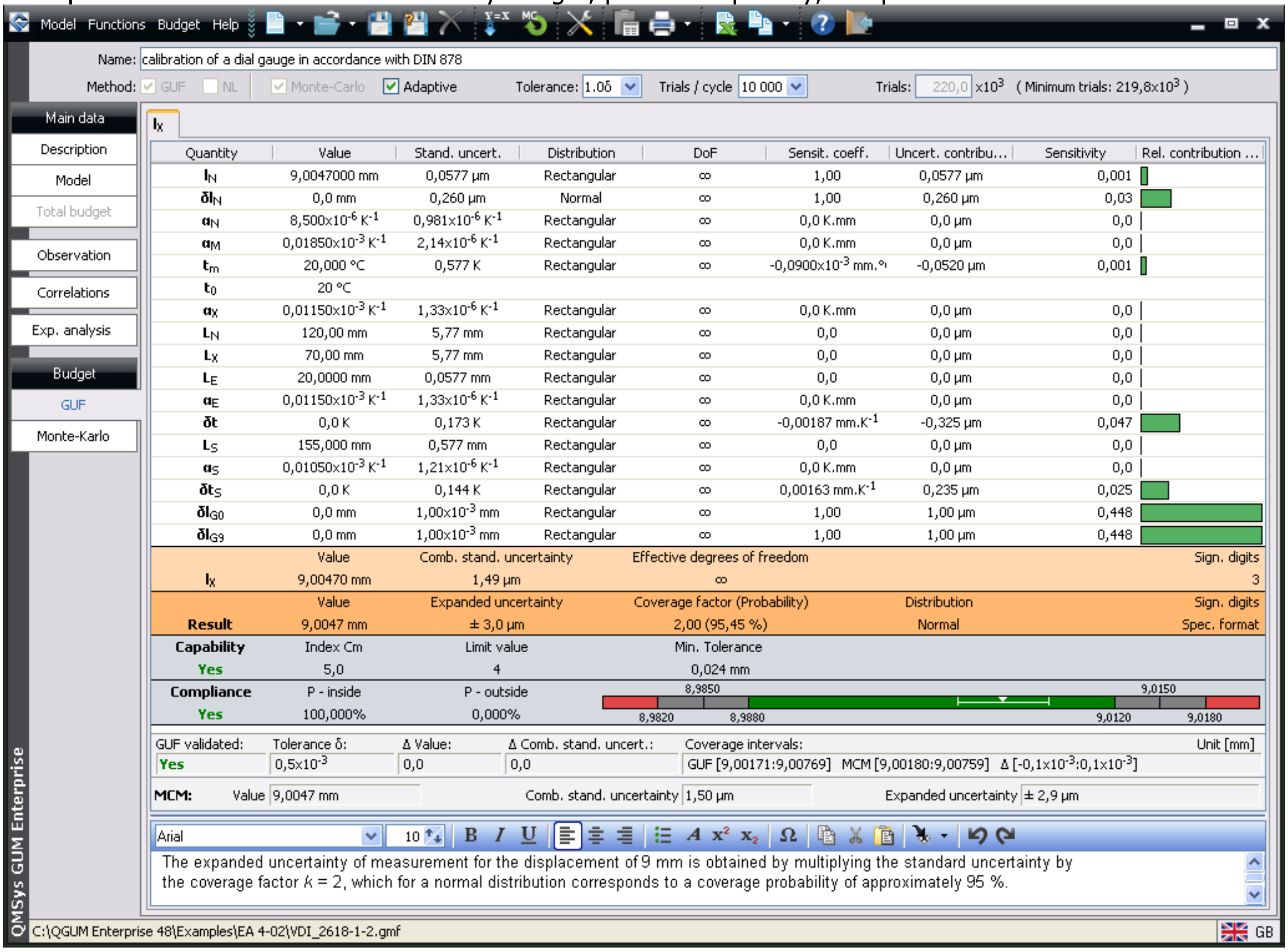
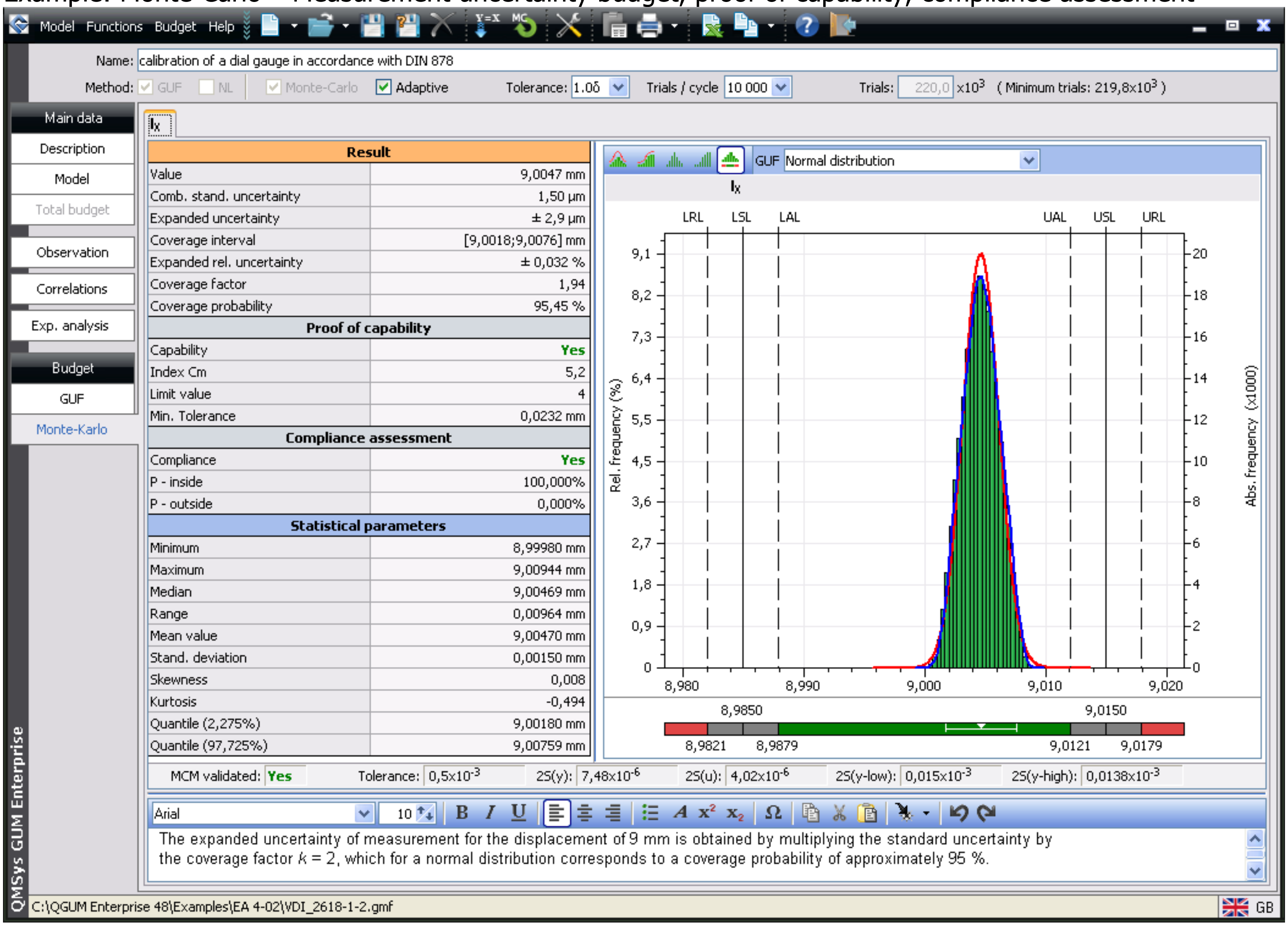
Example: GUF – Measurement uncertainty budget, proof of capability, compliance assessment



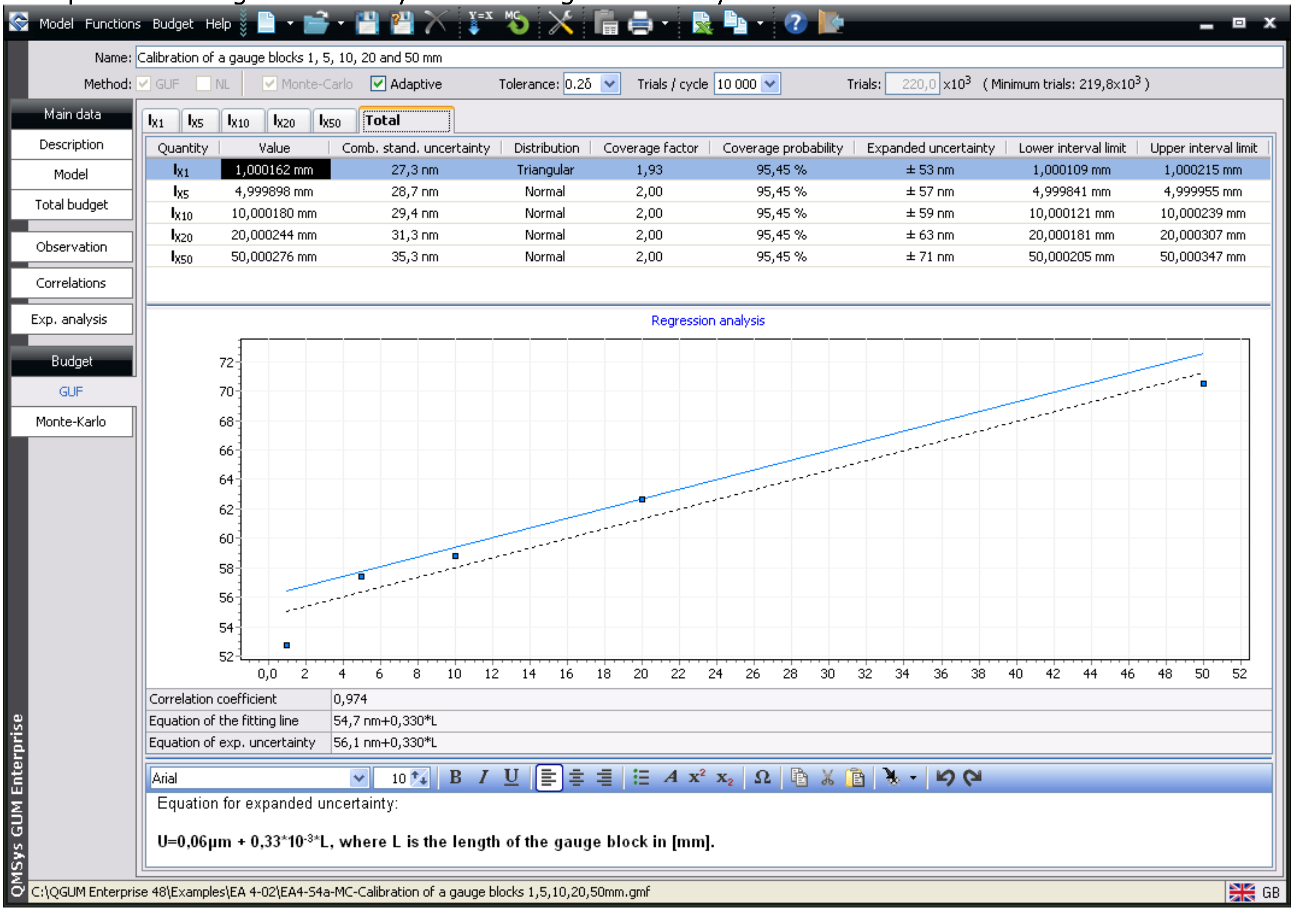
Example: Monte-Carlo – Measurement uncertainty budget, proof of capability, compliance assessment



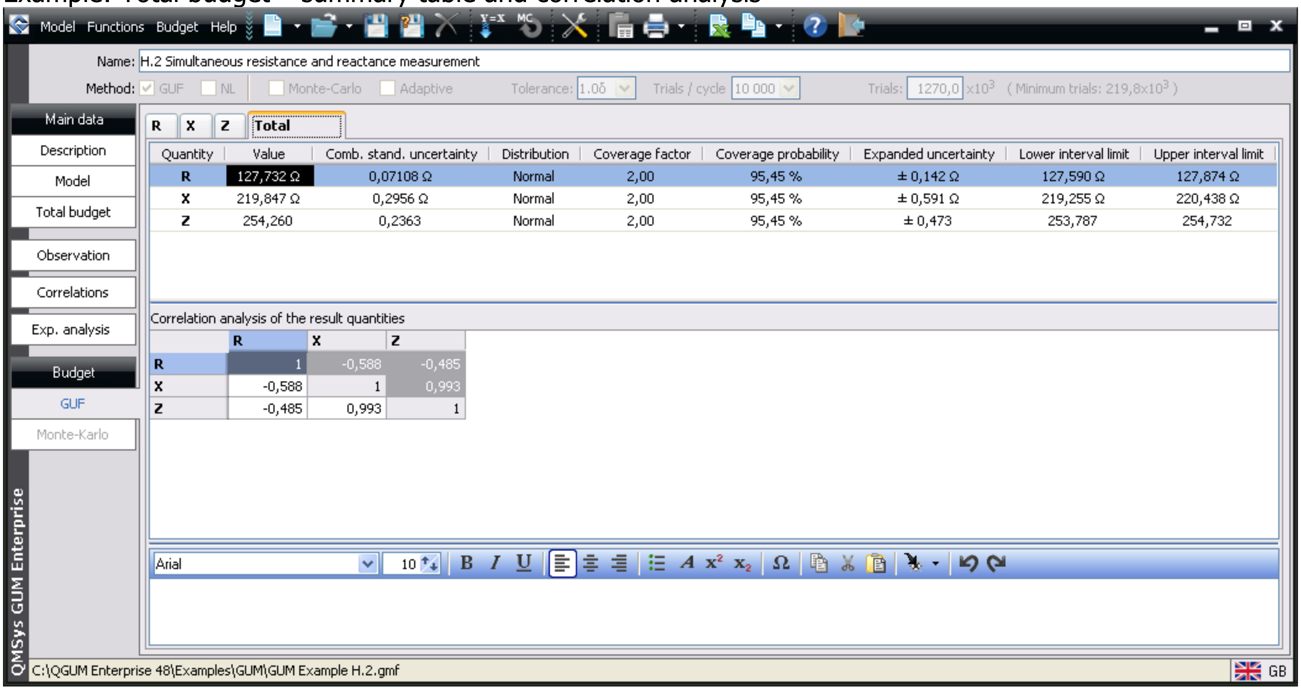
The *Total budget* offers the following additional analysis:

- Correlation analysis of the result quantities
- Regression analysis and calculation of the equation of the expanded measurement uncertainty for a certain measurement range.

Example: Total budget – summary table and regression analysis



Example: Total budget – summary table and correlation analysis



Each analysis can be completely saved in a file with a selectable name. In this way, the examination is available at any time for a later review or editing. Each saved analysis can be used as a starting point for new uncertainty analyses using the same model, but with new and changed data.

Appendix A: Validation of the QMSys GUM Software

ISO/IEC Guide 98-3:2008 (GUM:1995) Guide to the expression of uncertainty in measurement

Example	GUM				QMSys GUM, GUF Method				Comment / Distribution
	Estimate	u	k	U	Estimate	u	k	U	
H.1	50,000838 mm	32 nm	2,92	±92 nm	50,000838 mm	32 nm	2,92	±92 nm	t-distribution, f=16
H.2 (R)	127,732 Ω	0,071 Ω	2,00	±0,142 Ω	127,732 Ω	0,0711 Ω	2,00	±0,142 Ω	Normal
H.2 (X)	219,847 Ω	0,295 Ω	2,00	±0,590 Ω	219,847 Ω	0,2956 Ω	2,00	±0,591 Ω	Normal
H.2 (Z)	254,260 Ω	0,236 Ω	2,00	±0,472 Ω	254,260 Ω	0,2363 Ω	2,00	±0,473 Ω	Normal
H.2 Korrel.	r(V,I) = -0,36 ; r(V,φ) = 0,86 ; r(I,φ) = -0,65 r(R,X) = -0,588 ; r(R,Z) = -0,485 ; r(X,Z) = 0,993				r(V,I) = -0,355 ; r(V,φ) = 0,858 ; r(I,φ) = -0,645 r(R,X) = -0,588 ; r(R,Z) = -0,485 ; r(X,Z) = 0,993				Input quantities Result quantities
H.3	-0,1494 °C	0,0041 °C	2,00	±0,0082 °C	-0,1494 °C	0,00414 °C	2,00	±0,0083 °C	Normal
H.4	0,430 Bq/g	0,0083 Bq/g	2,00	±0,017 Bq/g	0,430 Bq/g	0,00833 Bq/g	2,00	±0,017 Bq/g	Normal

ISO Guide 98-3/S.1, JCGM 101 Suppl. 1 to the "GUM" - Propagation of distributions using a Monte-Carlo method

Example	JCGM 101:2008				QMSys GUM				Comment / GUF-Distribution
	Method	Estimate	u	Cover. interval	Method	Estimate	u	Cover. interval	
9.2.2	MCM	0,00	2.00	[-3,92; 3,92]	MCM	0,00	2.00	[-3,92; 3,92]	
9.2.3	MCM	0,00	2.00	[-3,88; 3,88]	MCM	0,00	2.00	[-3,88; 3,88]	
9.2.4	MCM	0,00	10.1	[-17,0; 17,0]	MCM	0,00	10.1	[-17,0; 17,0]	
	GUF	0,00	10.1	[-19,9; 19,9]	GUF	0,00	10.1	[-17,1; 17,1]	Trapezoidal, ß=0,74
9.3	MCM	1,2341	0,0754	[1,0834; 1,3825]	MCM	1,2340	0,0754	[1,0845; 1,3835]	
	GUF2	1,2340	0,0750	[1,0870; 1,3810]	GUF-NL	1,2340	0,0754	[1,0862; 1,3818]	Normal
9.4.2.2	MCM	50	50	[0; 150]	MCM	50	50	[0; 150]	x 10^{-6} / Normal
	Analytical	50	50	-	GUF-NL	50	50	[-48; 148]	x 10^{-6} / Normal
9.4.2.3	MCM	150	112	[0; 367]	MCM	150	112	[0; 366]	x 10^{-6} / Normal
	Analytical	150	112	-	GUF-NL	150	112	[-69; 369]	x 10^{-6} / Normal
9.4.2.4	MCM	2551	502	[1590; 3543]	MCM	2551	503	[1591; 3547]	x 10^{-6} / Normal
	Analytical	2550	502	-	GUF-NL	2550	502	[1565; 3535]	x 10^{-6} / Normal
9.4.3.2.1	MCM	50	67	[0; 185]	MCM	50	67	[0; 185]	x 10^{-6} / Normal
	Analytical	50	67	-	GUF-NL	50	67	[-81; 181]	x 10^{-6} / Normal
9.4.3.2.2	MCM	150	121	[13; 398]	MCM	150	120	[13; 398]	x 10^{-6} / Normal
	Analytical	150	121	-	GUF-NL	150	120	[-86; 386]	x 10^{-6} / Normal
9.4.3.2.3	MCM	2551	504	[1628; 3555]	MCM	2550	505	[1629; 3561]	x 10^{-6} / Normal
	Analytical	2550	505	-	GUF-NL	2550	505	[1561; 3539]	x 10^{-6} / Normal
9.5	MCM	838 nm	36 nm	[745; 932] nm	MCM	838 nm	36 nm	[744; 932] nm	
	Analytical	-	-	-	GUF-NL	838 nm	36 nm	[746; 930] nm	Normal
	GUF	838 nm	32 nm	[745; 931] nm	GUF	838 nm	32 nm	[745; 931] nm	t-distribution, f=16

EA-4/02 , DAkkS-DKD-3 Expression of the Uncertainty of Measurement in Calibration

Example	DKD-3, EA-4/02				QMSys GUM, GUF Method				Comment / Distribution
	Estimate	u	k	U	Estimate	u	k	U	
S.2	10000,025 g	0,0293 g	2,00	±0,059 g	10000,025 g	0,0293 g	2,00	±0,059 g	Normal
S.3	10000,178 Ω	0,00833 Ω	2,00	±0,017 Ω	10000,178 Ω	0,00833 Ω	2,00	±0,017 Ω	Normal
S.4	49,999926 mm	34,3 nm	2,00	±69 nm	49,999926 mm	34,3 nm	2,00	±69 nm	Normal
S.5 (t_x)	1000,5 °C	0,641 K	2,00	±1,3 K	1000,5 °C	0,641 K	2,00	±1,3 K	Normal
S.5 (V_x)	36229 µV	25.0 µV	2,00	±50 µV	36229 µV	24.8 µV	2,00	±50 µV	Normal
S.6	0,933	0,0162	2,00	±0,032	0,933	0,0162	2,00	±0,032	Normal
S.7	30,043 dB	0,0224 dB	2,00	±0,045 dB	30,043 dB	0,0224 dB	2,00	±0,045 dB	Normal
S.9	0,10 V	0,030 V	1,65	±0,05 V	0,10 V	0,030 V	1,65	±0,05 V	Rectangular
S.10	0,10 mm	0,033 mm	1,83	±0,06 mm	0,100 mm	0,0323 mm	1,84	±0,060 mm	Trapezoidal, ß=0,33
S.11	180,1 °C	0,164 K	1,81	±0,3 K	180,10 °C	0,164 K	1,80	±0,30 K	Trapezoidal, ß=0,43
S.12 (V_x)	199,95 l	0,109 l	-	-	199,95 l	0,109 l	2,00	±0,22 l	Normal
S.12 (e_x)	0,0003	0,00068	-	-	0,0002	0,000681	2,00	±0,0014 l	Normal
S.12 (e_{xav})	0,001	0,91 x10^{-3}	2,28	±0,002	0,001	0,910 x10^{-3}	2.28	±0,0021	t-distribution, f=10
S.13	90,00025 mm	0,414 µm	2,00	±0,9 µm	90,00024 mm	0,411 µm	2,00	±0,82 µm	Normal

EURACHEM/CITAC Guide CG 4 Quantifying Uncertainty in Analytical Measurement

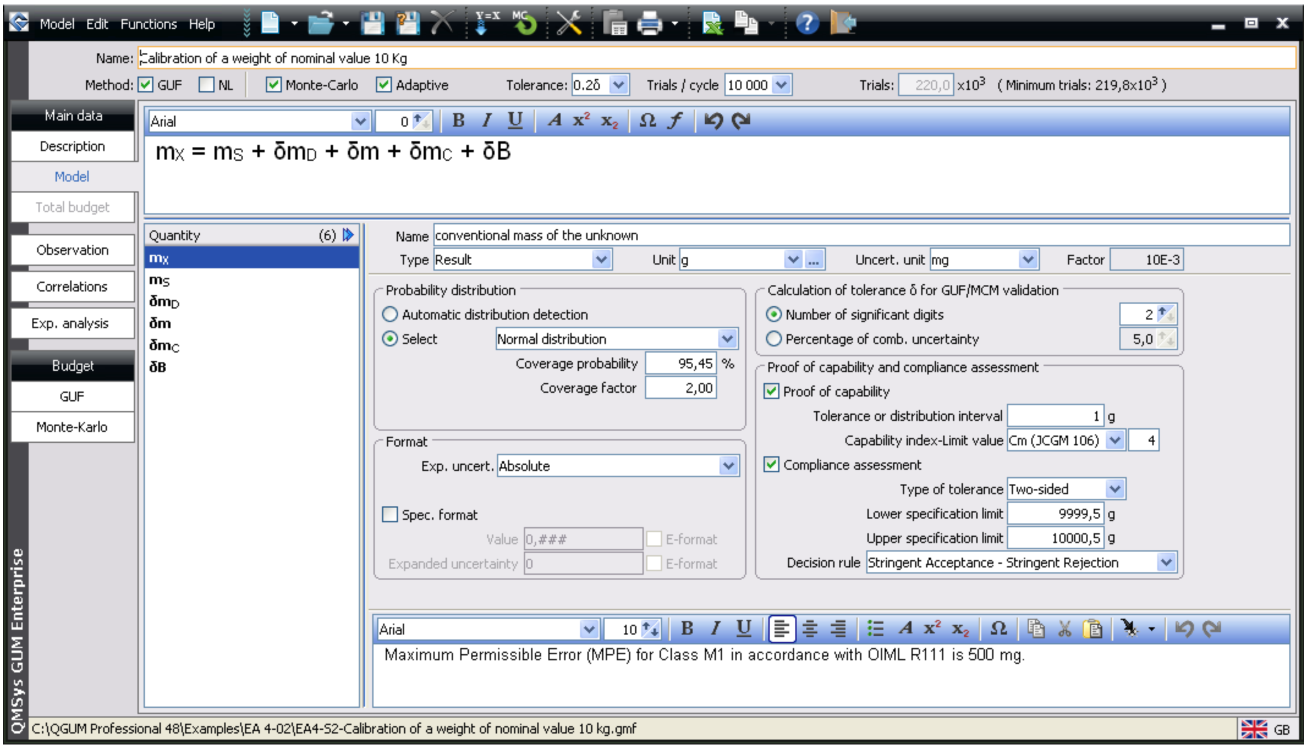
Example	EURACHEM/CITAC Guide CG 4			QMSys GUM, GUF Method, Normal distribution, k = 2.00		
	Estimate	u	U	Estimate	u	U
A.1	1002,7 mg.l^{-1}	0,9 mg.l^{-1}	±1,8 mg.l^{-1}	1002,7 mg.l^{-1}	0,835 mg.l^{-1}	±1,7 mg.l^{-1}
A.2	0,1021 mol.l^{-1}	0,00010 mol.l^{-1}	0,0002 mol.l^{-1}	0,10214 mol.l^{-1}	0,000101 mol.l^{-1}	0,00020 mol.l^{-1}
A.3	0,1014 mol.l^{-1}	0,00018 mol.l^{-1}	0,0004 mol.l^{-1}	0,10139 mol.l^{-1}	0,000184 mol.l^{-1}	0,00037 mol.l^{-1}
A.3 repl.	0,1014 mol.l^{-1}	0,00016 mol.l^{-1}	0,0003 mol.l^{-1}	0,10139 mol.l^{-1}	0,000165 mol.l^{-1}	0,00033 mol.l^{-1}
A.4	1,11	0,34	0,68	1,11	0,339	0,68
A.5	0,036 mg.dm^{-2}	0,0034 mg.dm^{-2}	0,007 mg.dm^{-2}	0,0363 mg.dm^{-2}	0,00342 mg.dm^{-2}	0,0068 mg.dm^{-2}
A.7	0,05370 µmol.q^{-1}	0,00018 µmol.q^{-1}	0,00036 µmol.q^{-1}	0,05374 µmol.q^{-1}	0,000180 µmol.q^{-1}	0,00036 µmol.q^{-1}

u - Combined standard uncertainty; U - Expanded uncertainty; k - Coverage factor; f - Degrees of freedom;
ß - Shape factor

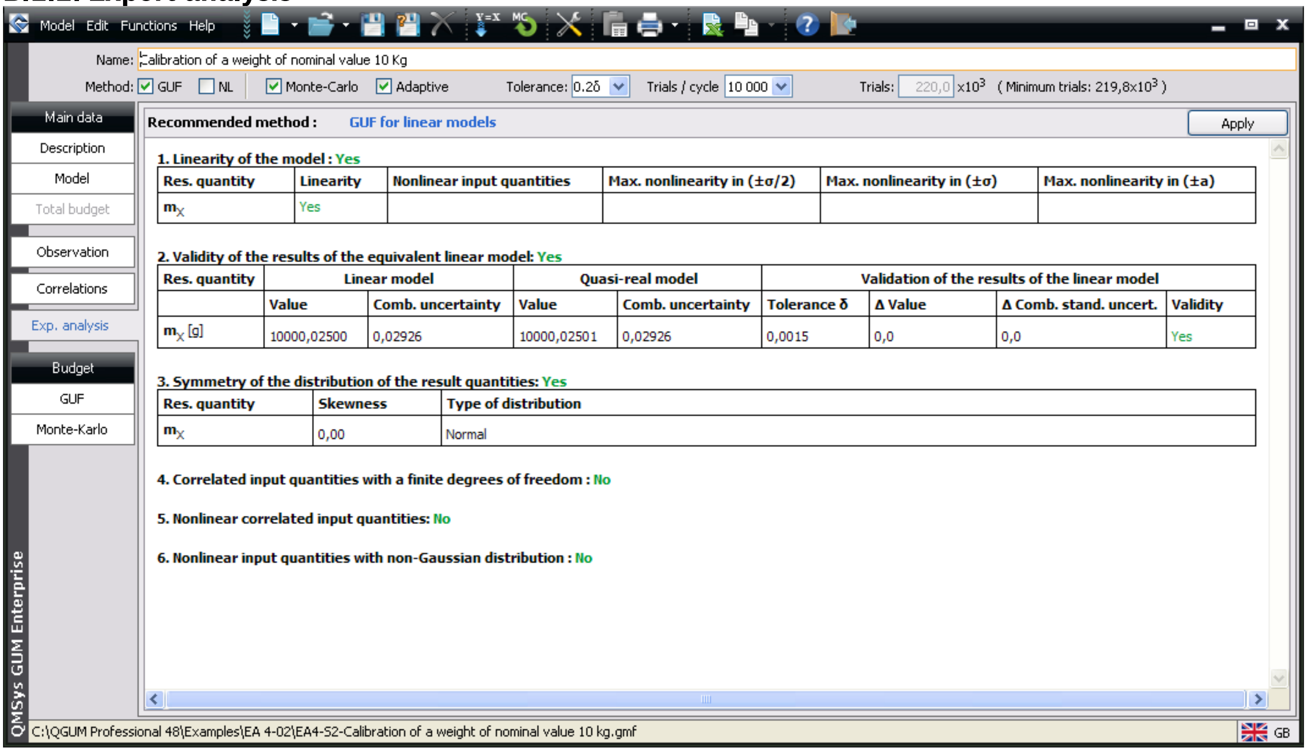
Appendix B: Examples

B.1. Linear model, result quantity with normal distribution (EA-4/02 - S2)

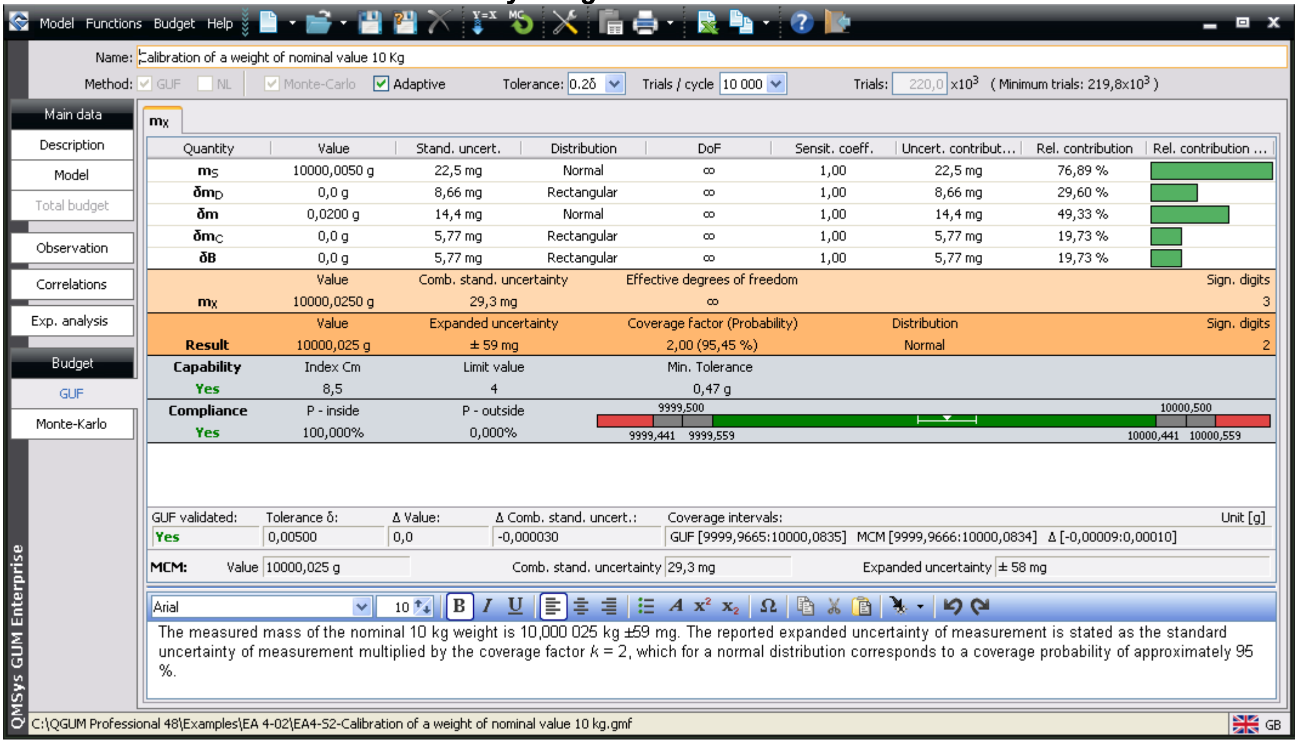
B.1.1. Model



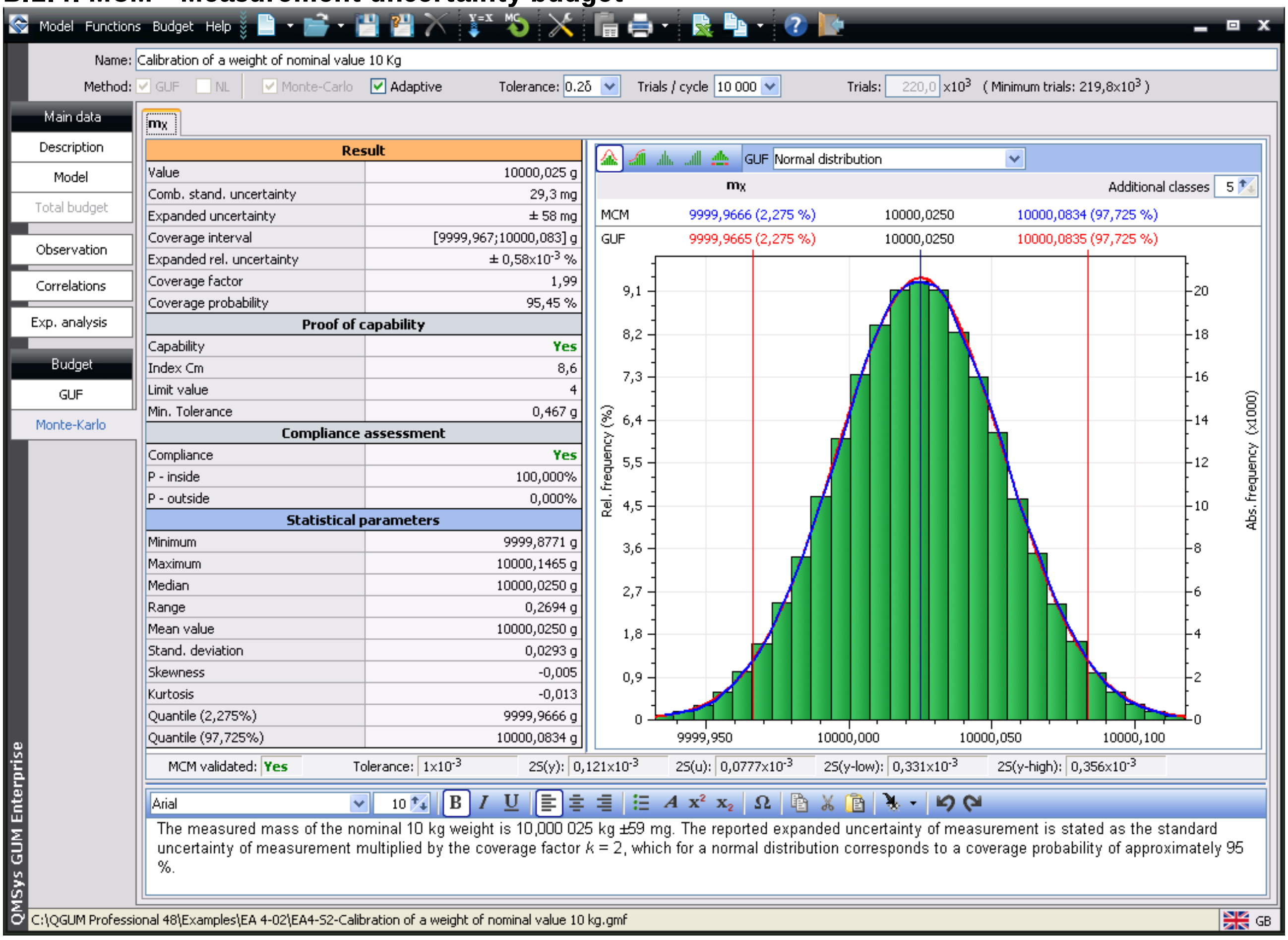
B.1.2. Expert analysis



B.1.3. GUF – Measurement uncertainty budget



B.1.4. MCM - Measurement uncertainty budget



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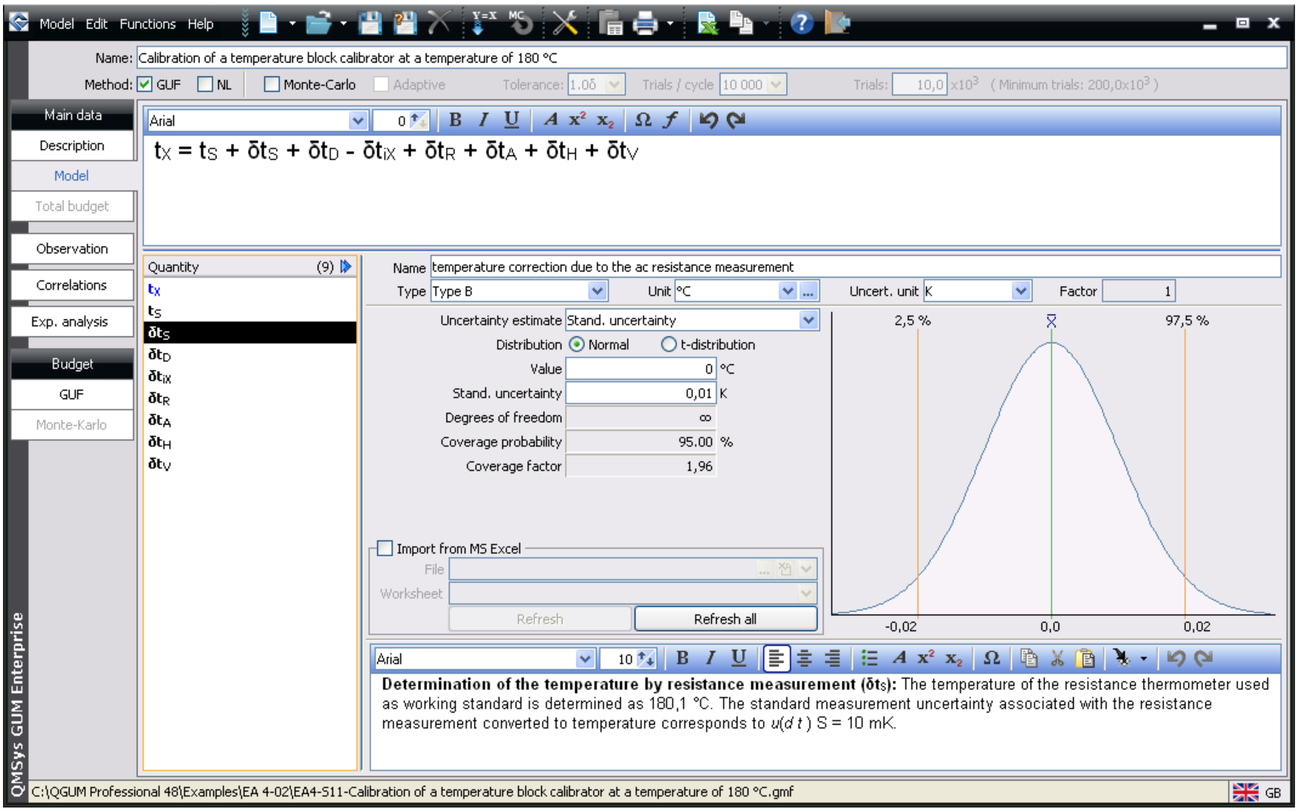
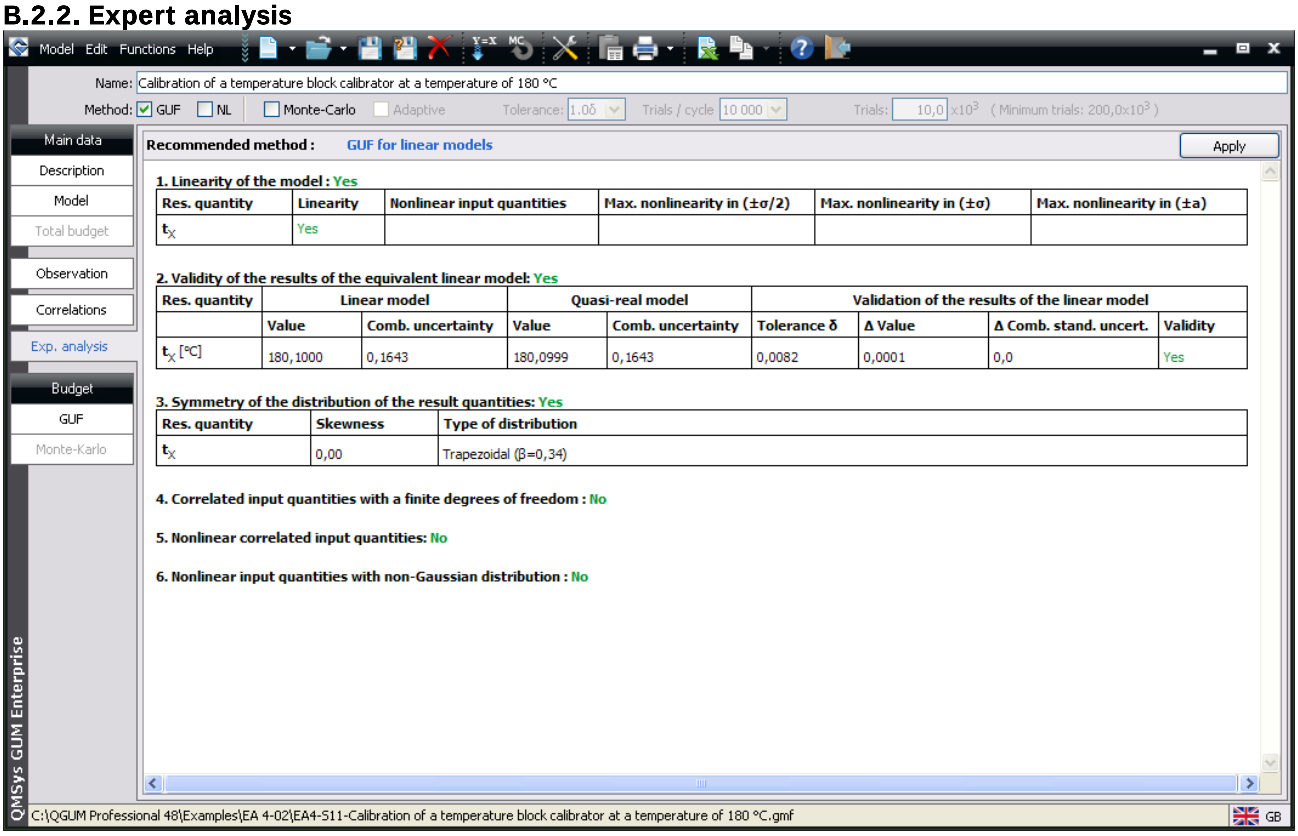
B.2. Linear model, result quantity with trapezoidal distribution (EA-4/02 – S11)

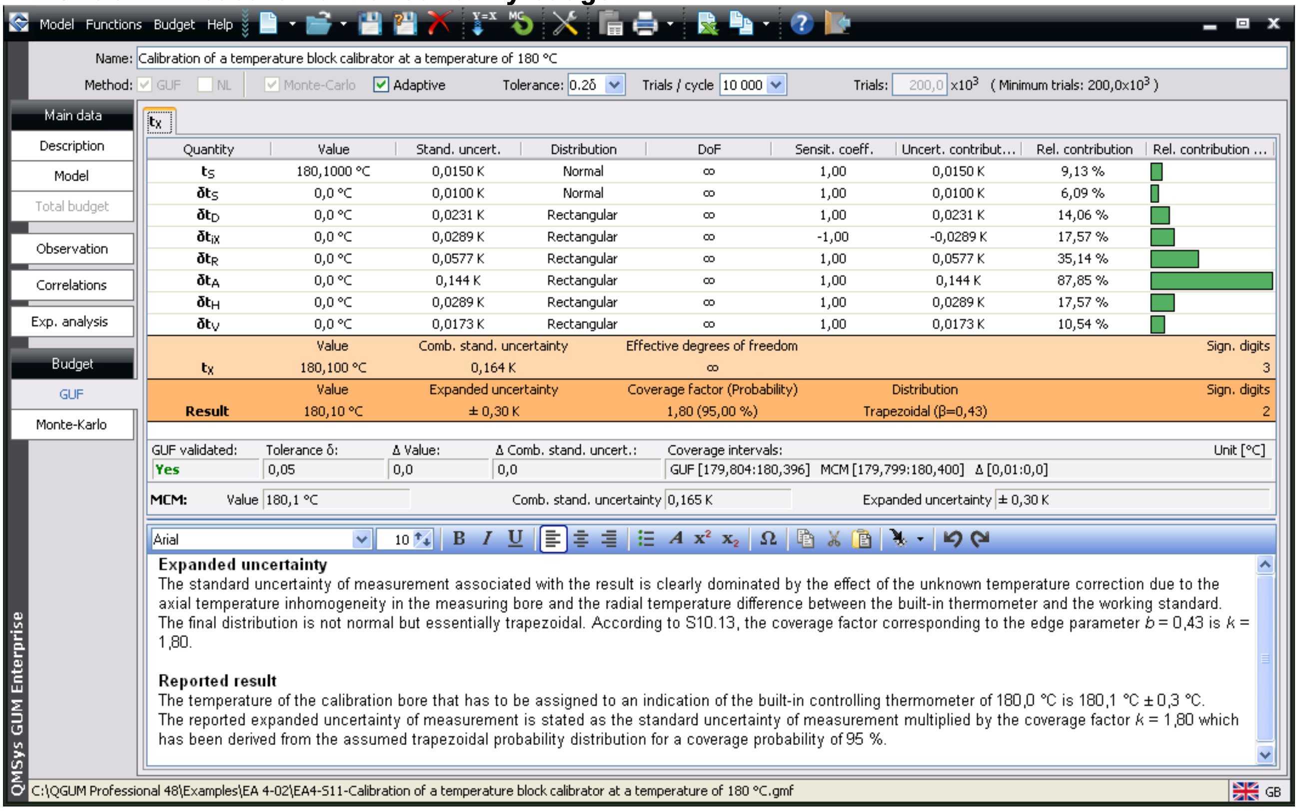
B.2.1. Model



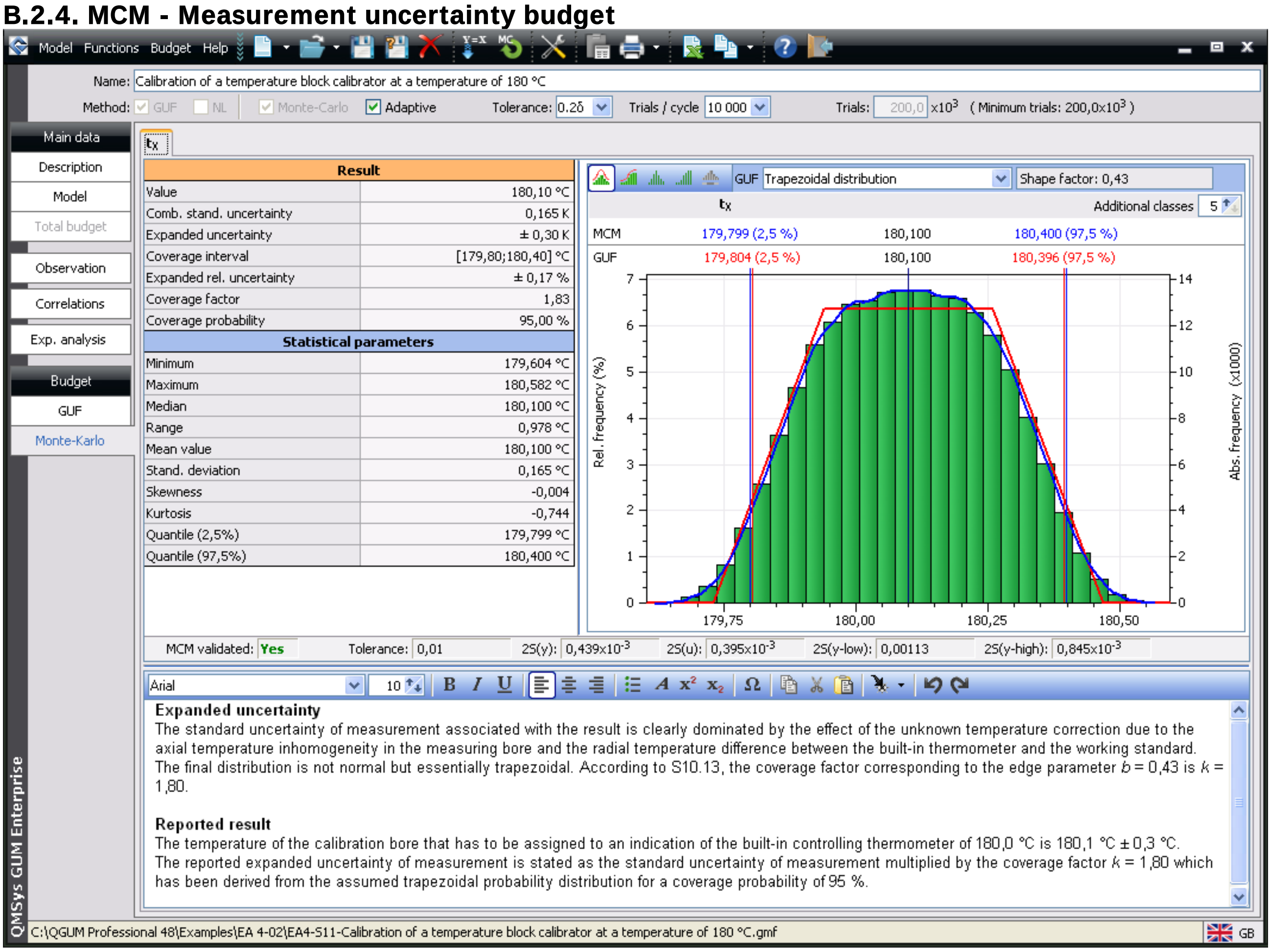
B.2.2. Expert analysis



B.2.3. GUF – Measurement uncertainty budget

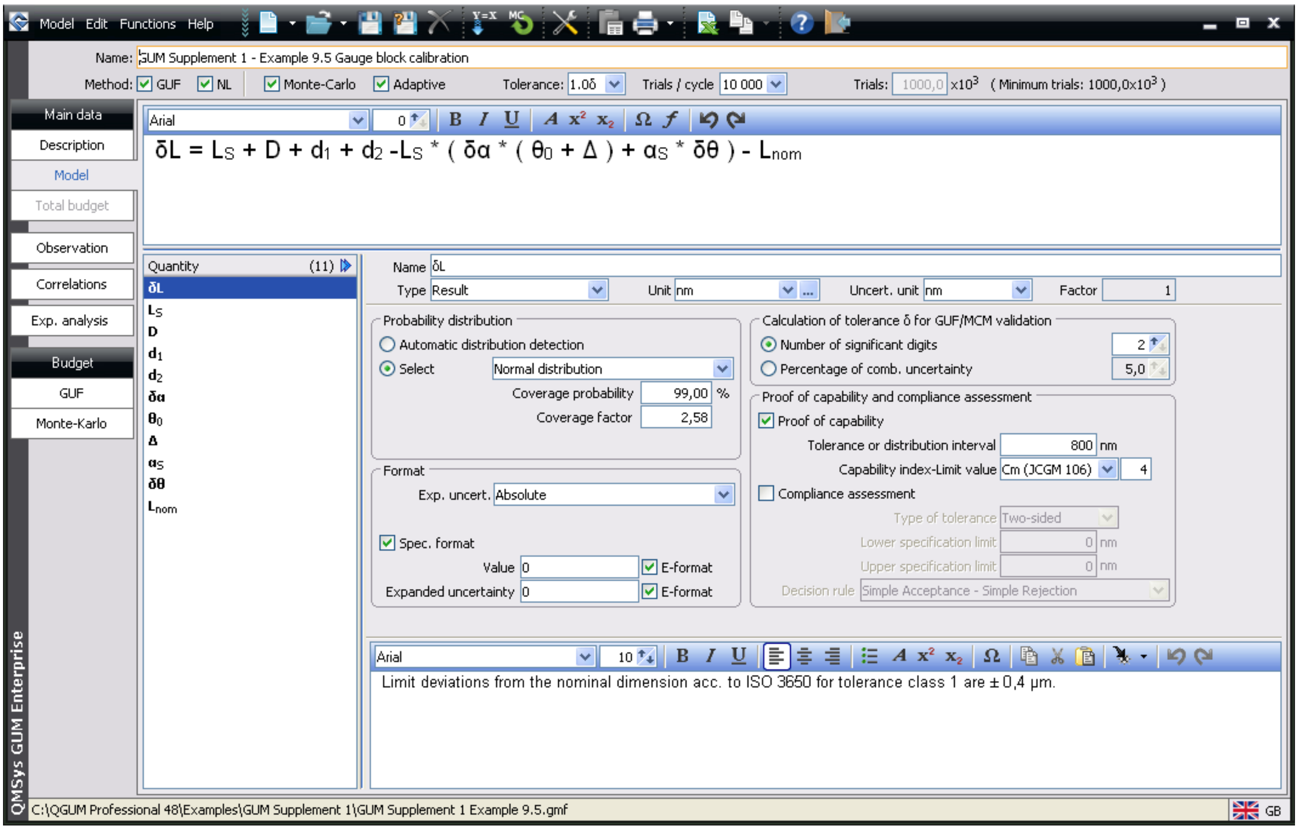


B.2.4. MCM - Measurement uncertainty budget

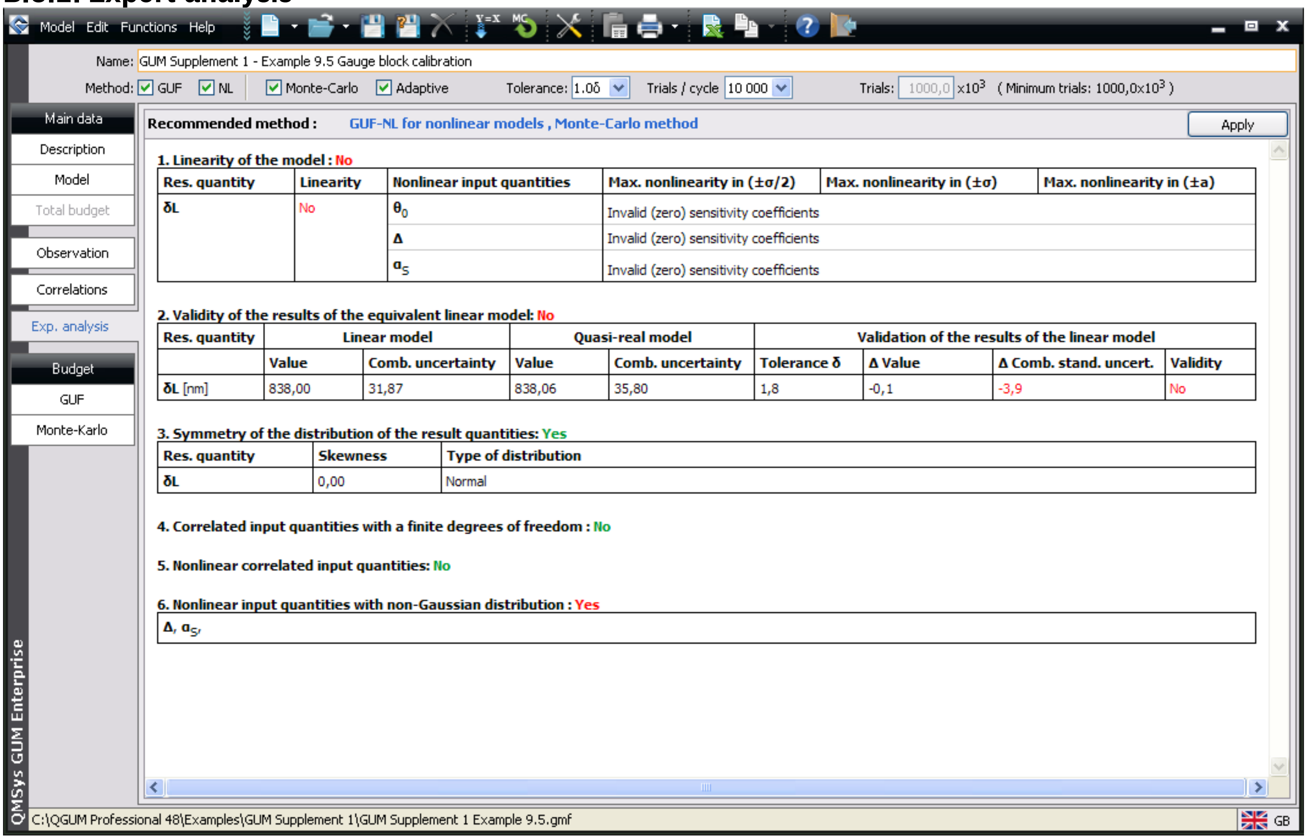


B.3. Non-linear model, result quantity with normal distribution (GUM Suppl. 1, 9.5)

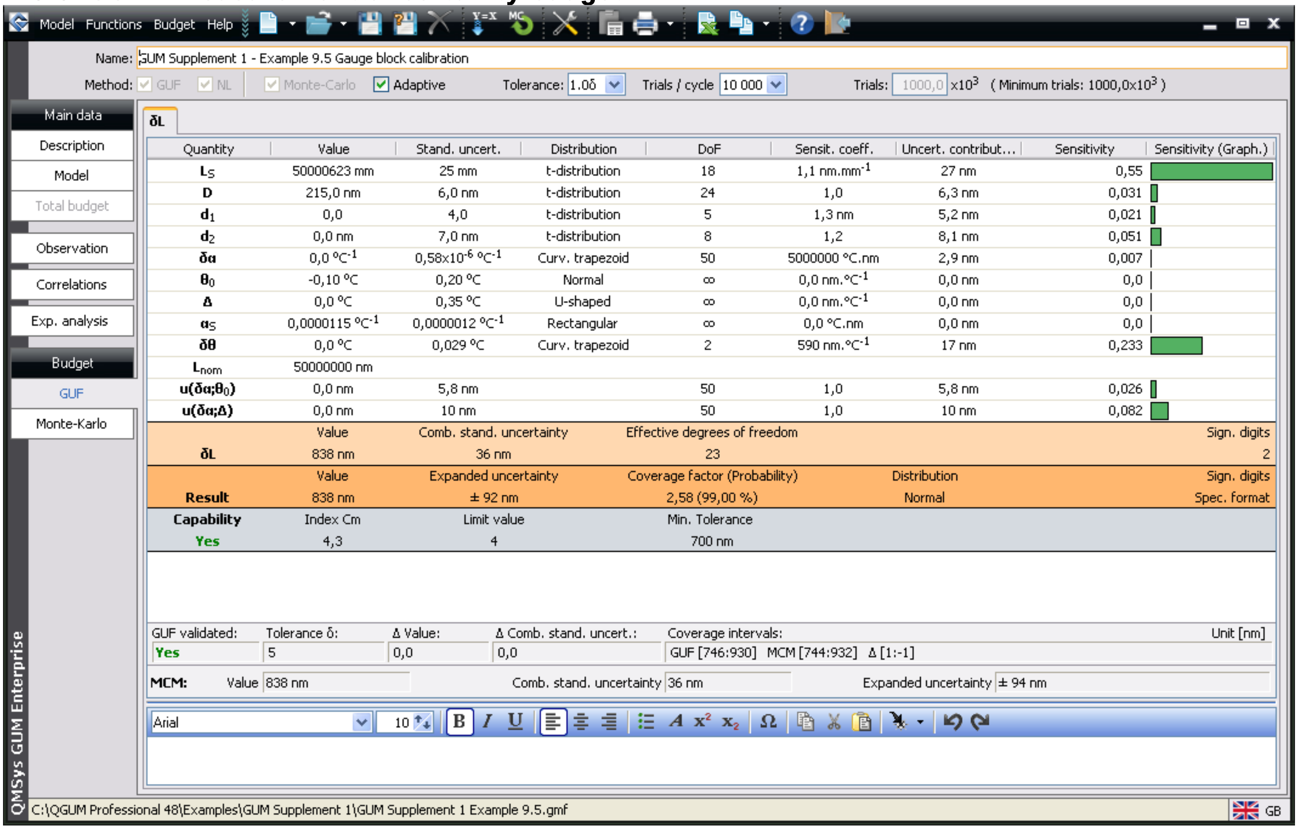
B.3.1. Model



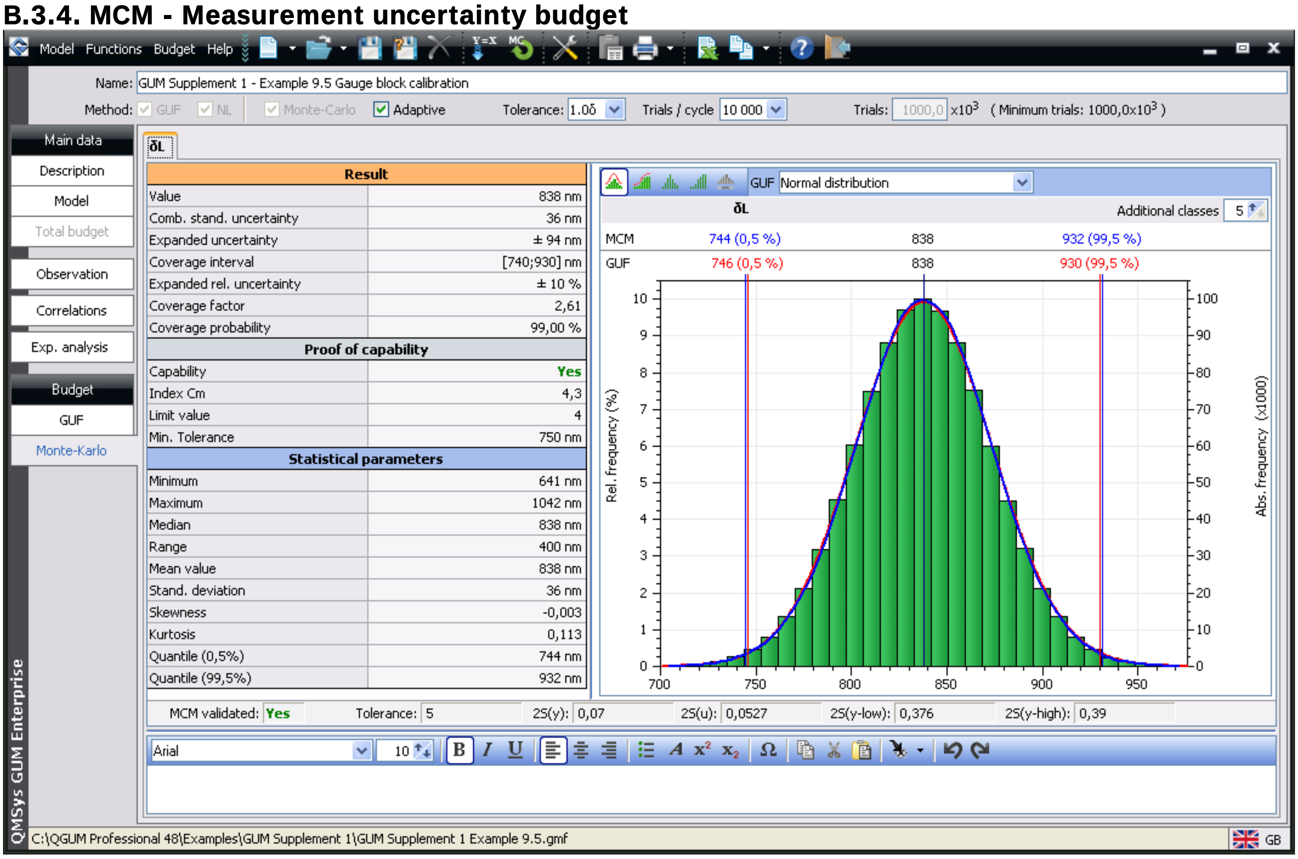
B.3.2. Expert analysis



B.3.3. GUF – Measurement uncertainty budget

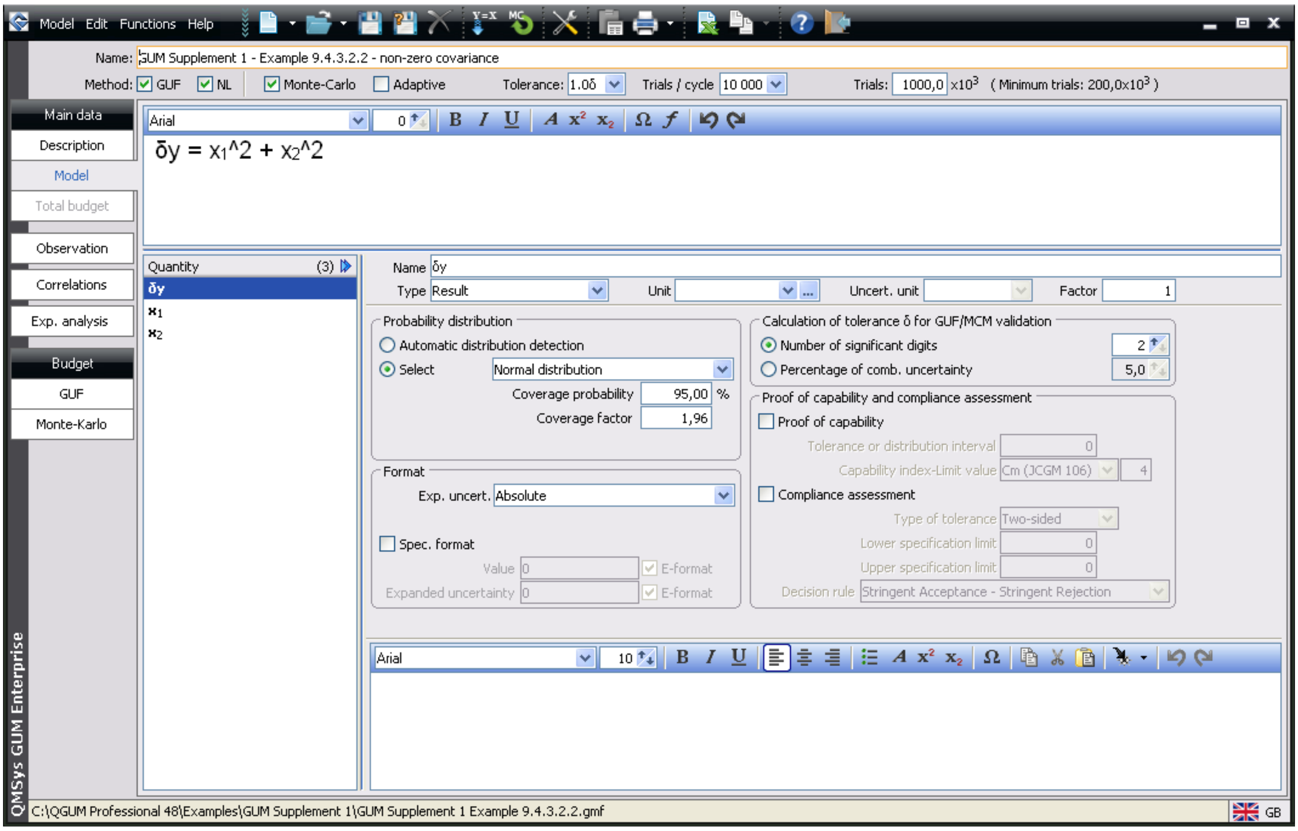


B.3.4. MCM - Measurement uncertainty budget

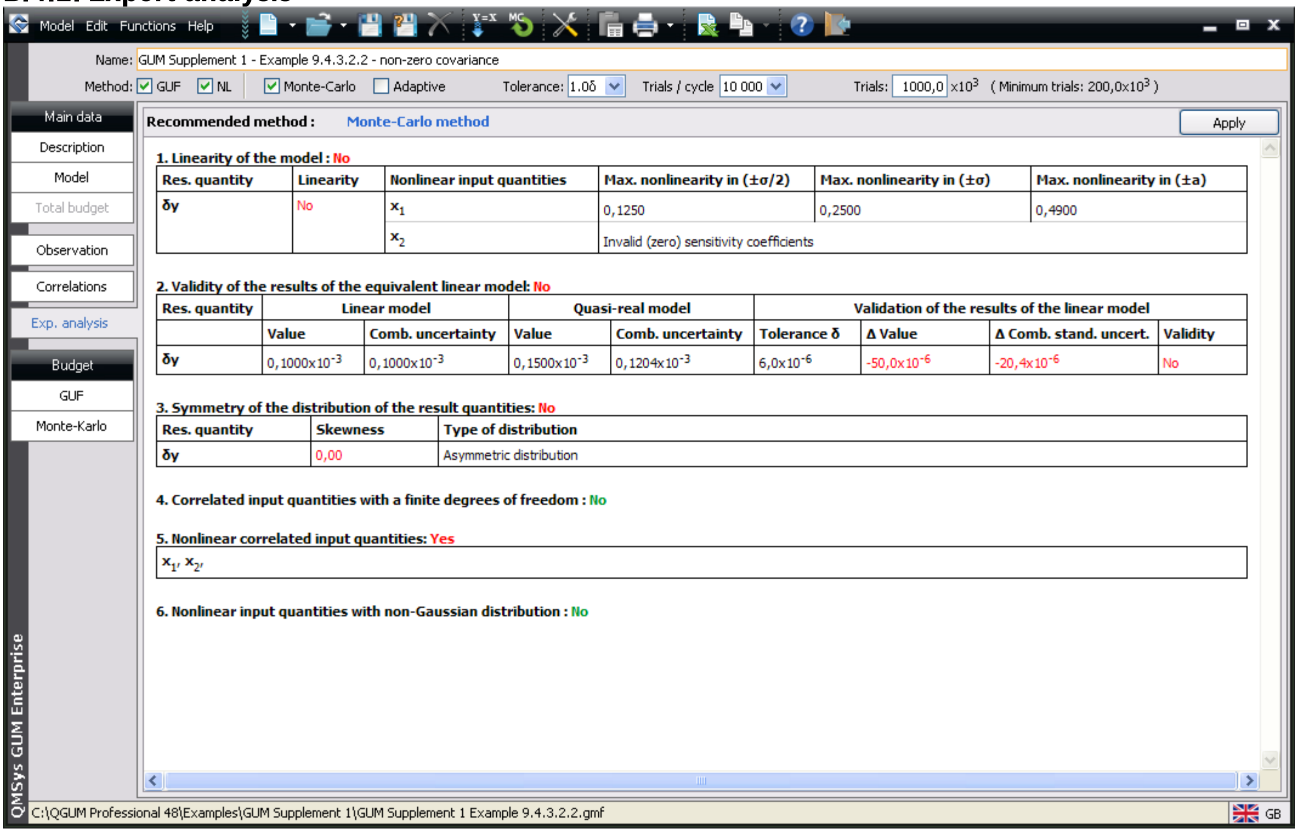


B.4. Non-linear model, result quantity with asymmetric distribution (GUM Suppl. 1, 9.4.3.2.2, correlation coefficient = 0,9)

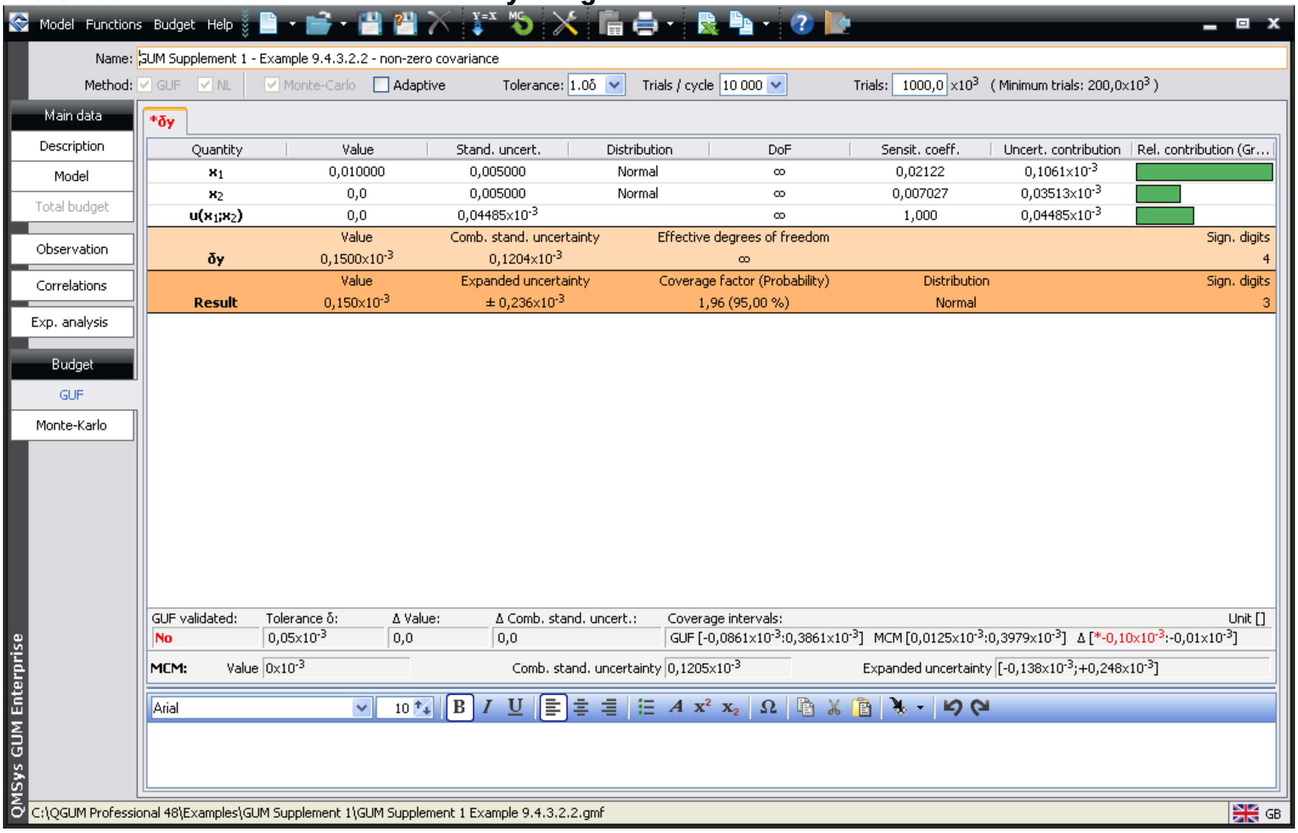
B.4.1. Model



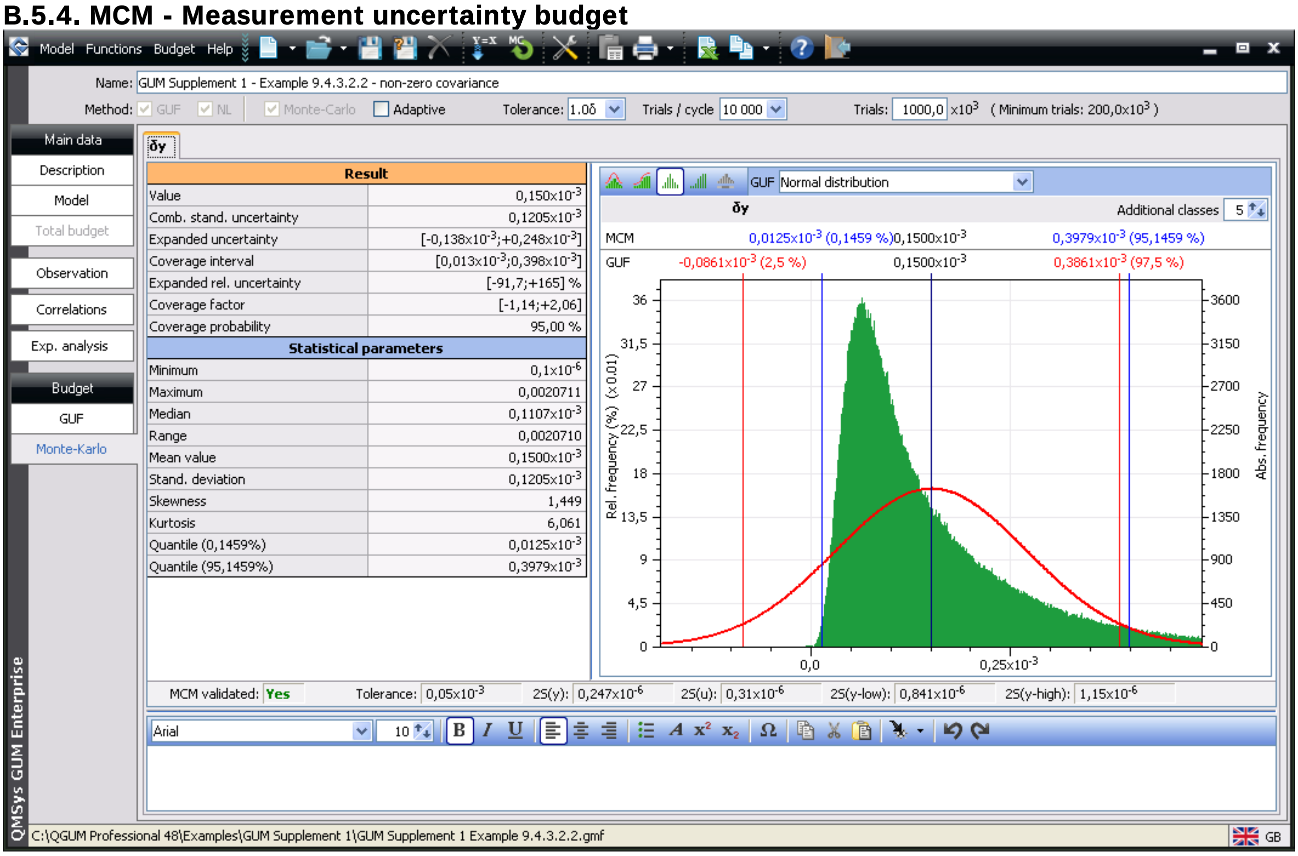
B.4.2. Expert analysis



B.5.3. GUF – Measurement uncertainty budget

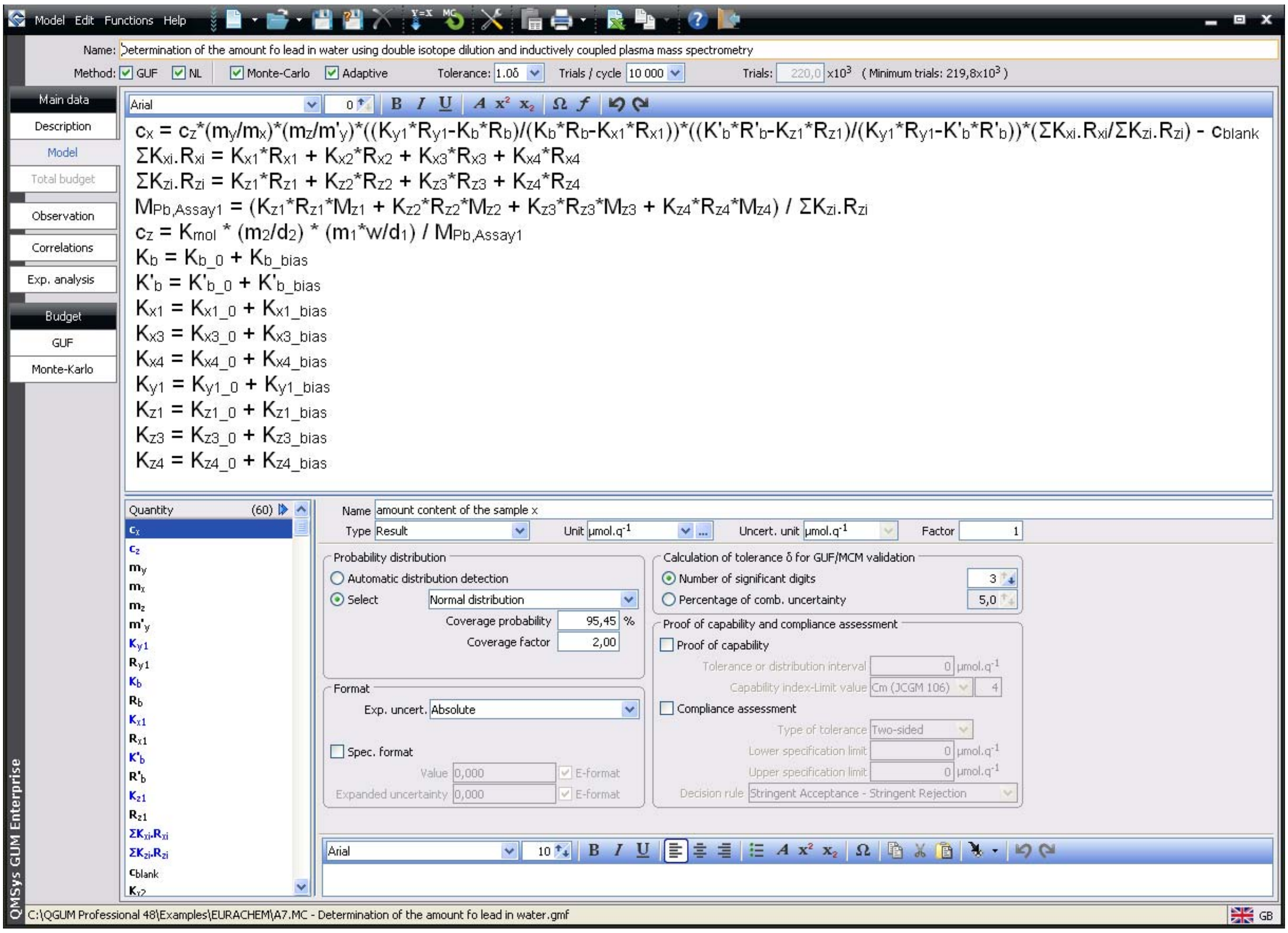


B.5.4. MCM - Measurement uncertainty budget

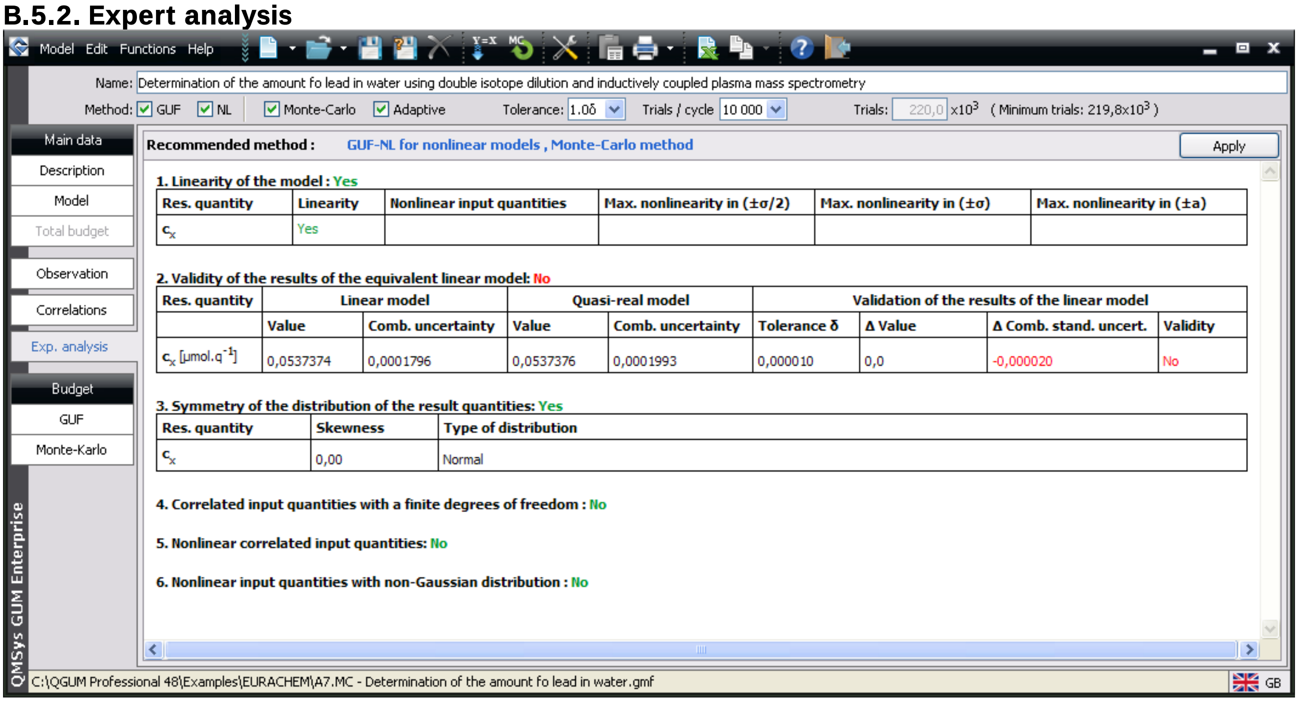


B.5. Non-linear model, result quantity with normal distribution, no relevant interaction of multiple input quantities (EURACHEM/CITAC CG 4, A.7)

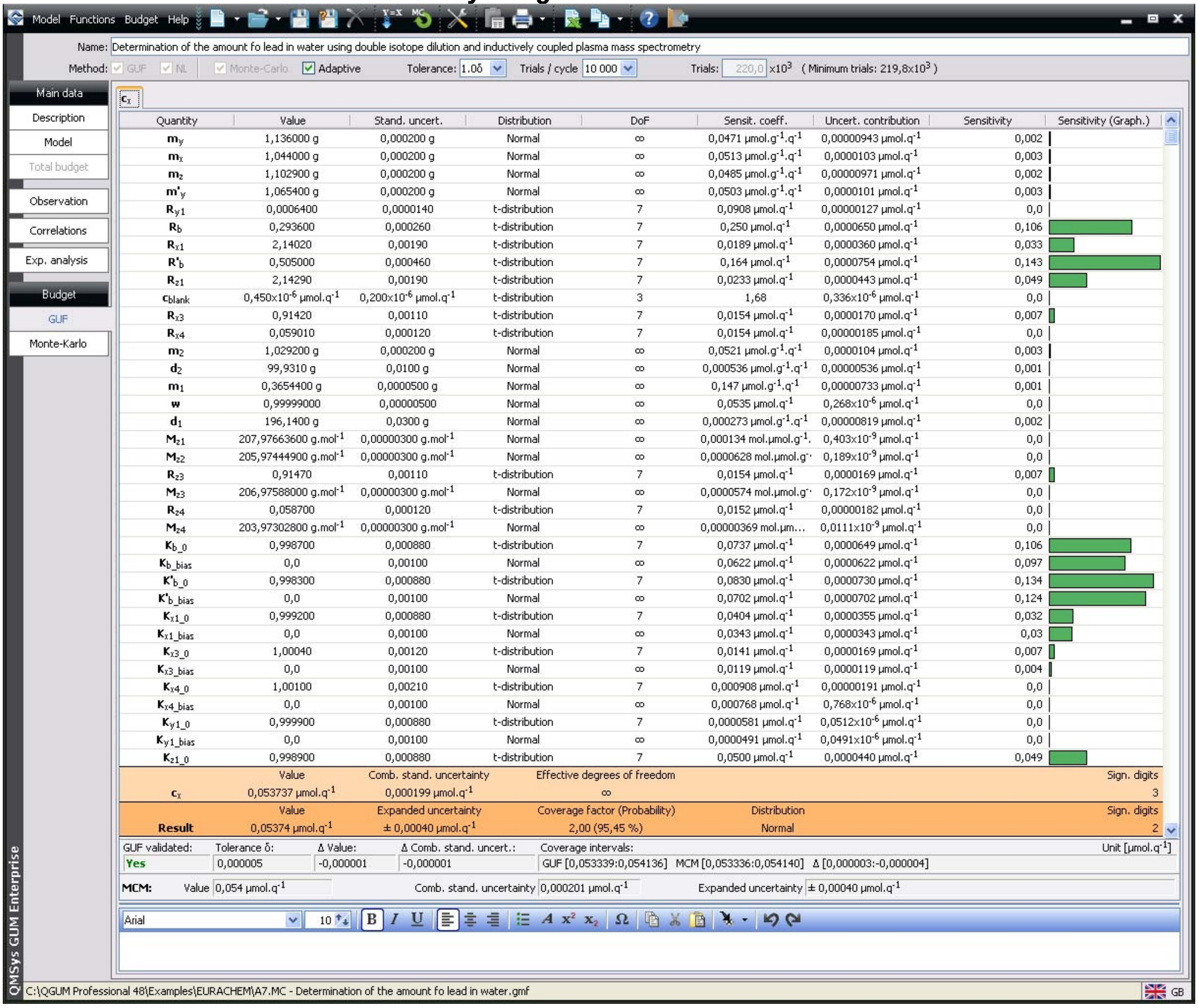
B.5.1. Model



B.5.2. Expert analysis



B.5.3. GUF – Measurement uncertainty budget



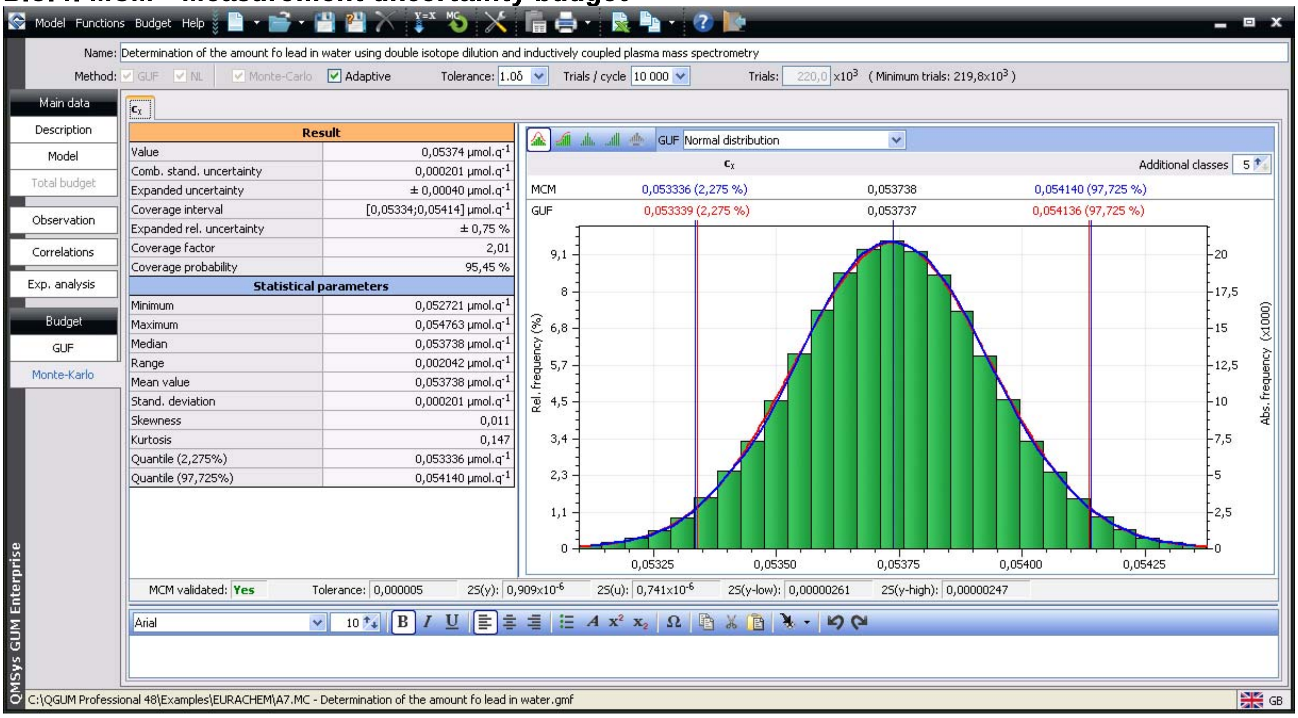
Quantity	Value	Stand. uncert.	Distribution	DoF	Sensit. coeff.	Uncert. contribution	Sensitivity
m_y	1,136000 g	0,000200 g	Normal	∞	0,0471 µmol.g⁻¹.q⁻¹	0,00000943 µmol.q⁻¹	0,002
m_x	1,044000 g	0,000200 g	Normal	∞	0,0513 µmol.g⁻¹.q⁻¹	0,0000103 µmol.q⁻¹	0,003
m_z	1,102900 g	0,000200 g	Normal	∞	0,0485 µmol.g⁻¹.q⁻¹	0,00000971 µmol.q⁻¹	0,002
m'_y	1,065400 g	0,000200 g	Normal	∞	0,0503 µmol.g⁻¹.q⁻¹	0,0000101 µmol.q⁻¹	0,003
R_{y1}	0,0006400	0,0000140	t-distribution	7	0,0908 µmol.q⁻¹	0,00000127 µmol.q⁻¹	0,0
R_b	0,293600	0,000260	t-distribution	7	0,250 µmol.q⁻¹	0,0000650 µmol.q⁻¹	0,106
R_{x1}	2,14020	0,00190	t-distribution	7	0,0189 µmol.q⁻¹	0,0000360 µmol.q⁻¹	0,033
R'_b	0,505000	0,000460	t-distribution	7	0,164 µmol.q⁻¹	0,0000754 µmol.q⁻¹	0,143
R_{z1}	2,14290	0,00190	t-distribution	7	0,0233 µmol.q⁻¹	0,0000443 µmol.q⁻¹	0,049
c_{blank}	0,450x10⁻⁶ µmol.q⁻¹	0,200x10⁻⁶ µmol.q⁻¹	t-distribution	3	1,68	0,336x10⁻⁶ µmol.q⁻¹	0,0
R_{x3}	0,91420	0,00110	t-distribution	7	0,0154 µmol.q⁻¹	0,0000170 µmol.q⁻¹	0,007
R_{x4}	0,059010	0,000120	t-distribution	7	0,0154 µmol.q⁻¹	0,00000185 µmol.q⁻¹	0,0
m_2	1,029200 g	0,000200 g	Normal	∞	0,0521 µmol.g⁻¹.q⁻¹	0,0000104 µmol.q⁻¹	0,003
d_2	99,9310 g	0,0100 g	Normal	∞	0,000536 µmol.g⁻¹.q⁻¹	0,00000536 µmol.q⁻¹	0,001
m_1	0,3654400 g	0,0000500 g	Normal	∞	0,147 µmol.g⁻¹.q⁻¹	0,00000733 µmol.q⁻¹	0,001
w	0,99999000	0,00000500	Normal	∞	0,0535 µmol.q⁻¹	0,268x10⁻⁶ µmol.q⁻¹	0,0
d_1	196,1400 g	0,0300 g	Normal	∞	0,000273 µmol.g⁻¹.q⁻¹	0,00000819 µmol.q⁻¹	0,002
M_{z1}	207,97663600 g.mol⁻¹	0,00000300 g.mol⁻¹	Normal	∞	0,000134 mol.µmol.g⁻¹.	0,403x10⁻⁹ µmol.q⁻¹	0,0
M_{z2}	205,97444900 g.mol⁻¹	0,00000300 g.mol⁻¹	Normal	∞	0,0000628 mol.µmol.g⁻·	0,189x10⁻⁹ µmol.q⁻¹	0,0
R_{z3}	0,91470	0,00110	t-distribution	7	0,0154 µmol.q⁻¹	0,0000169 µmol.q⁻¹	0,007
M_{z3}	206,97588000 g.mol⁻¹	0,00000300 g.mol⁻¹	Normal	∞	0,0000574 mol.µmol.g⁻·	0,172x10⁻⁹ µmol.q⁻¹	0,0
R_{z4}	0,058700	0,000120	t-distribution	7	0,0152 µmol.q⁻¹	0,00000182 µmol.q⁻¹	0,0
M_{z4}	203,97302800 g.mol⁻¹	0,00000300 g.mol⁻¹	Normal	∞	0,00000369 mol.µm...	0,0111x10⁻⁹ µmol.q⁻¹	0,0
K_{b_0}	0,998700	0,000880	t-distribution	7	0,0737 µmol.q⁻¹	0,0000649 µmol.q⁻¹	0,106
K_{b_bias}	0,0	0,00100	Normal	∞	0,0622 µmol.q⁻¹	0,0000622 µmol.q⁻¹	0,097
K'_{b_0}	0,998300	0,000880	t-distribution	7	0,0830 µmol.q⁻¹	0,0000730 µmol.q⁻¹	0,134
K'_{b_bias}	0,0	0,00100	Normal	∞	0,0702 µmol.q⁻¹	0,0000702 µmol.q⁻¹	0,124
K_{x1_0}	0,999200	0,000880	t-distribution	7	0,0404 µmol.q⁻¹	0,0000355 µmol.q⁻¹	0,032
K_{x1_bias}	0,0	0,00100	Normal	∞	0,0343 µmol.q⁻¹	0,0000343 µmol.q⁻¹	0,03
K_{x3_0}	1,00040	0,00120	t-distribution	7	0,0141 µmol.q⁻¹	0,0000169 µmol.q⁻¹	0,007
K_{x3_bias}	0,0	0,00100	Normal	∞	0,0119 µmol.q⁻¹	0,0000119 µmol.q⁻¹	0,004
K_{x4_0}	1,00100	0,00210	t-distribution	7	0,000908 µmol.q⁻¹	0,00000191 µmol.q⁻¹	0,0
K_{x4_bias}	0,0	0,00100	Normal	∞	0,000768 µmol.q⁻¹	0,768x10⁻⁶ µmol.q⁻¹	0,0
K_{y1_0}	0,999900	0,000880	t-distribution	7	0,0000581 µmol.q⁻¹	0,0512x10⁻⁶ µmol.q⁻¹	0,0
K_{y1_bias}	0,0	0,00100	Normal	∞	0,0000491 µmol.q⁻¹	0,0491x10⁻⁶ µmol.q⁻¹	0,0
K_{z1_0}	0,998900	0,000880	t-distribution	7	0,0500 µmol.q⁻¹	0,0000440 µmol.q⁻¹	0,049

	Value	Comb. stand. uncertainty	Effective degrees of freedom		Sign. digits
c_x	0,053737 µmol.q⁻¹	0,000199 µmol.q⁻¹	∞		3
	Value	Expanded uncertainty	Coverage factor (Probability)	Distribution	Sign. digits
Result	0,05374 µmol.q⁻¹	± 0,00040 µmol.q⁻¹	2,00 (95,45 %)	Normal	2

GUF validated:	Tolerance δ:	Δ Value:	Δ Comb. stand. uncert.:	Coverage intervals:	Unit [µmol.q⁻¹]
Yes	0,000005	-0,000001	-0,000001	GUF [0,053339;0,054136] MCM [0,053336;0,054140] Δ [0,000003;-0,000004]	

MCM: Value 0,054 µmol.q⁻¹ Comb. stand. uncertainty 0,000201 µmol.q⁻¹ Expanded uncertainty ± 0,00040 µmol.q⁻¹

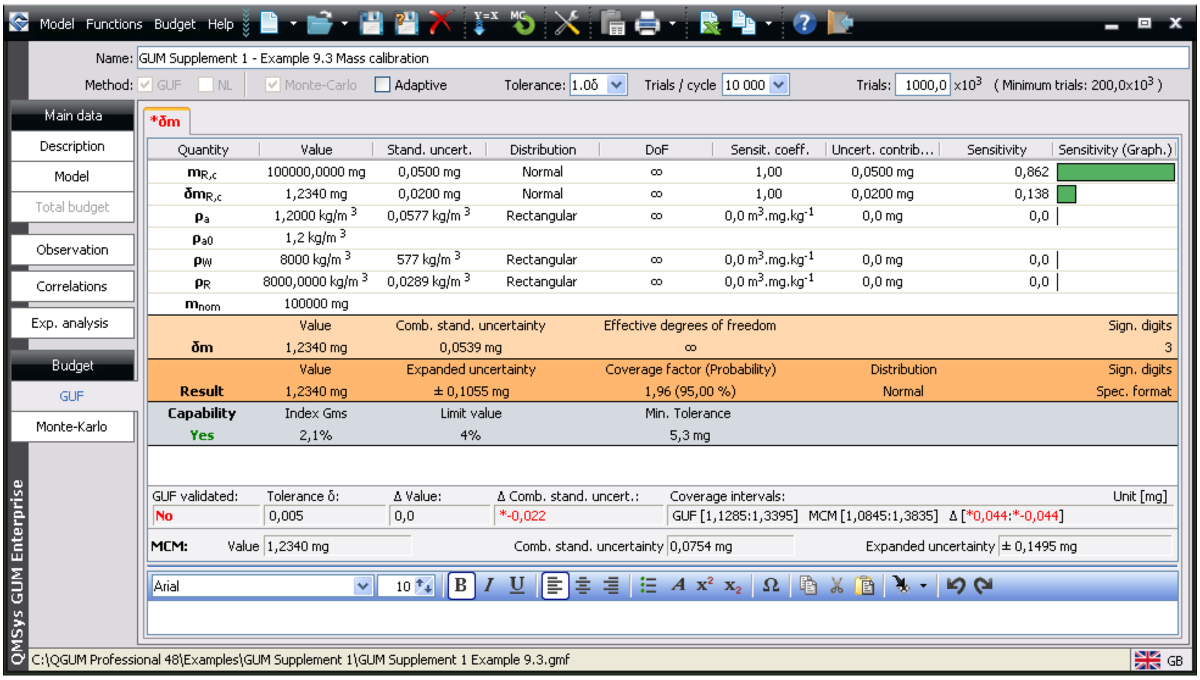
B.5.4. MCM - Measurement uncertainty budget



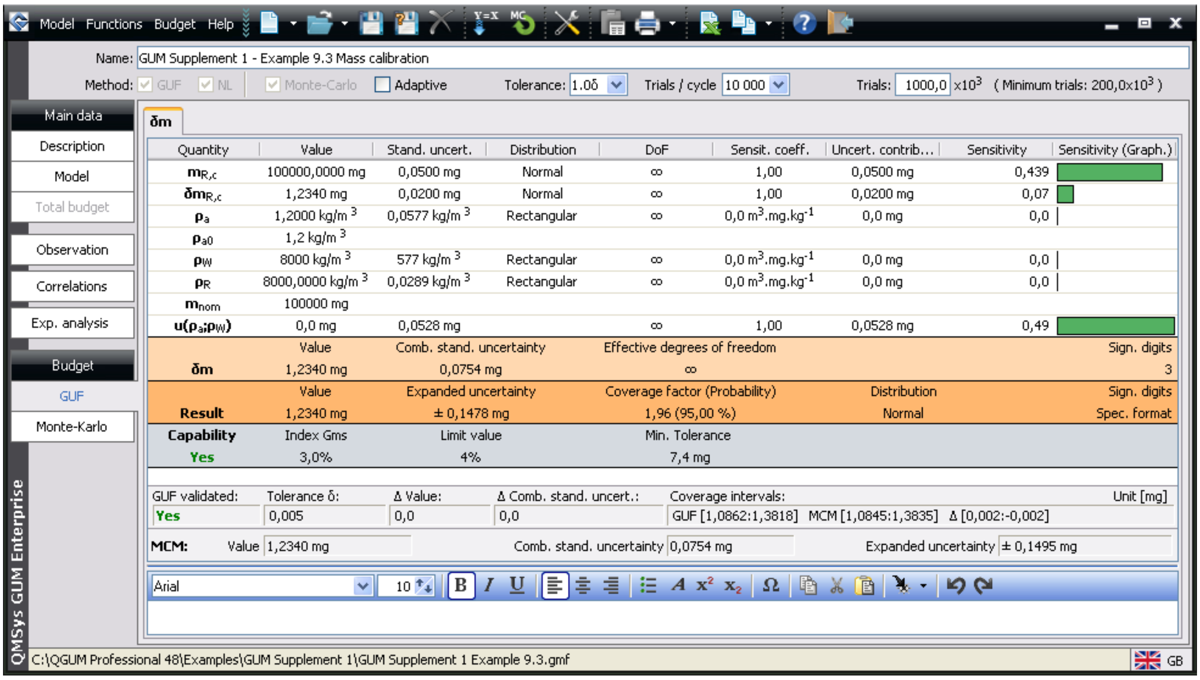
Result	
Value	0,05374 µmol.q⁻¹
Comb. stand. uncertainty	0,000201 µmol.q⁻¹
Expanded uncertainty	± 0,00040 µmol.q⁻¹
Coverage interval	[0,05334;0,05414] µmol.q⁻¹
Expanded rel. uncertainty	± 0,75 %
Coverage factor	2,01
Coverage probability	95,45 %
Statistical parameters	
Minimum	0,052721 µmol.q⁻¹
Maximum	0,054763 µmol.q⁻¹
Median	0,053738 µmol.q⁻¹
Range	0,002042 µmol.q⁻¹
Mean value	0,053738 µmol.q⁻¹
Stand. deviation	0,000201 µmol.q⁻¹
Skewness	0,011
Kurtosis	0,147
Quantile (2,275%)	0,053336 µmol.q⁻¹
Quantile (97,725%)	0,054140 µmol.q⁻¹

B.6. Comparison of the GUF - Methods (GUM S1, Example 9.3)

B.6.1. GUF – Method for linear Models

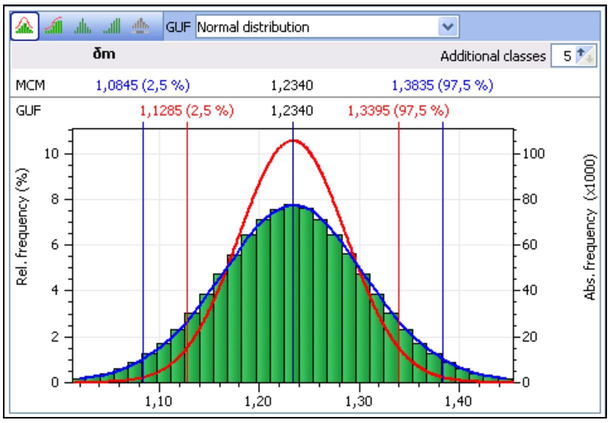


B.6.2. GUF-NL – Method for nonlinear Models



B.6.3. Comparison with the results of Monte-Carlo method

MCM – GUF for linear Models



MCM – GUF-NL for nonlinear Models